

# Mark Scheme (Final)

## Statistics 2 (6684) January 2009

GCE

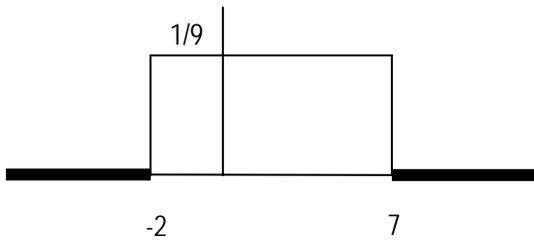
### GCE Mathematics (6684/01)

## General Marking Guidance

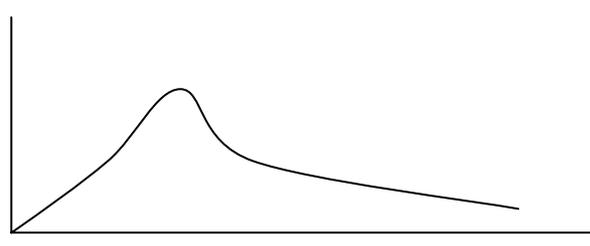
- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- If more than one response is seen and the candidate has not indicated which response they wish to submit then send the item to your Team Leader.

January 2009  
6684 Statistics 2  
Mark Scheme

Question Number	Scheme	Marks
1 .	The random variable $X$ is the number of daisies in a square. Poisson(3)	B1
(a)	$1 - P(X \leq 2) = 1 - 0.4232 \quad 1 - e^{-3}\left(1 + 3 + \frac{3^2}{2!}\right)$ $= 0.5768$	M1 A1 (3)
(b)	$P(X \leq 6) - P(X \leq 4) = 0.9665 - 0.8153 \quad e^{-3}\left(\frac{3^5}{5!} + \frac{3^6}{6!}\right)$ $= 0.1512$	M1 A1 (2)
(c)	$\mu = 3.69$ $\text{Var}(X) = \frac{1386}{80} - \left(\frac{295}{80}\right)^2$ $= 3.73/3.72/3.71 \quad \text{accept } s^2 = 3.77$	B1 M1 A1 (3)
(d)	For a Poisson model, Mean = Variance ; For these data $3.69 \approx 3.73$ $\Rightarrow$ Poisson model	B1 (1)
(e)	$\frac{e^{-3.6875} 3.6875^4}{4!} = 0.193$	M1 allow their mean or var Awrt 0.193 or 0.194 A1 ft (2)

Question Number	Scheme	Marks
2. (a)	$f(x) = \begin{cases} \frac{1}{9} & -2 \leq x \leq 7 \\ 0 & \text{otherwise} \end{cases}$	B1 B1 (2)
(b)		B1 B1 (2)
(c)	$E(X) = \underline{2.5} \quad \text{Var}(X) = \frac{1}{12}(7+2)^2 \text{ or } \underline{6.75}$ <p style="text-align: right;">both</p> $E(X^2) = \text{Var}(X) + E(X)^2$ $= 6.75 + 2.5^2$ $= 13$ <p><b>alternative</b></p> $\int_{-2}^7 x^2 f(x) dx = \left[ \frac{x^3}{27} \right]_{-2}^7$ $= 13$ <p style="text-align: right;"> <math>\int x^2 f(x)</math>                      attempt to integrate and use limits of -2 and 7                 </p>	B1 B1 (3) B1 M1 A1
(d)	$P(-0.2 < X < 0.6) = \frac{1}{9} \times 0.8$ $= \frac{4}{45} \text{ or } 0.0889 \quad \text{Or equiv}$ <p style="text-align: right;">awrt 0.089</p>	M1 A1 (2)

Question Number	Scheme	Marks
3.(a)	$X \sim B(20, 0.3)$ $P(X \leq 2) = 0.0355$ $P(X \geq 11) = 1 - 0.9829 = 0.0171$ Critical region is $(X \leq 2) \cup (X \geq 11)$	M1  A1 A1 (3)
(b)	Significance level = $0.0355 + 0.0171, = 0.0526$ or 5.26%	M1 A1 (2)
(c)	Insufficient evidence to reject $H_0$ <b>Or</b> sufficient evidence to accept $H_0$ /not significant $x = 3$ ( or the value) is not in the critical region or $0.1071 > 0.025$  Do not allow inconsistent comments	B1 ft B1 ft (2)

Question Number	Marks	Scheme
4.(a)	$\int_0^{10} kt dt = 1$ $\left[ \frac{kt^2}{2} \right]_0^{10} = 1$ $50k = 1$ $k = \frac{1}{50}$	<p>or Area of triangle = 1</p> <p>or <math>10 \times 0.5 \times 10k = 1</math> or linear equation in k</p> <p>cs0</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>(3)</p>
(b)	$\int_6^{10} kt dt = \left[ \frac{kt^2}{2} \right]_6^{10}$ $= \frac{16}{25}$	<p>M1</p> <p>A1</p> <p>(2)</p>
(c)	$E(T) = \int_0^{10} kt^2 dt = \left[ \frac{kt^3}{3} \right]_0^{10}$ $= 6\frac{2}{3}$ $\text{Var}(T) = \int_0^{10} kt^3 dt - \left(6\frac{2}{3}\right)^2 = \left[ \frac{kt^4}{4} \right]_0^{10} - \left(6\frac{2}{3}\right)^2$ $= 50 - \left(6\frac{2}{3}\right)^2$ $= 5\frac{5}{9}$	<p>M1</p> <p>A1</p> <p>M1;M1dep</p> <p>A1</p> <p>(5)</p>
(d)	10	B1
(e)		<p>B1</p> <p>(1)</p> <p>(1)</p>

Question Number	Scheme	Marks
5.(a)	<p><math>X</math> represents the number of defective components.</p> $P(X = 1) = (0.99)^9 (0.01) \times 10 = 0.0914$	M1A1 (2)
(b)	$P(X \geq 2) = 1 - P(X \leq 1)$ $= 1 - (p)^{10} - (a)$ $= 0.0043$	M1 A1√ A1 (3)
(c)	$X \sim \text{Po}(2.5)$ $P(1 \leq X \leq 4) = P(X \leq 4) - P(X = 0)$ $= 0.8912 - 0.0821$ $= 0.809$ <p>Normal distribution used. B1 for mean only</p>	B1B1 M1 A1 (4)
<p>Special case for parts a and b If they use 0.1 do not treat as misread as it makes it easier.</p> <p>(a) M1 A0 if they have 0.3874 (b) M1 A1ft A0 they will get 0.2639 (c) Could get B1 B0 M1 A0</p>		
<p>For any other values of <math>p</math> which are in the table do not use misread. Check using the tables. They could get (a) M1 A0 (b) M1 A1ft A0 (c) B1 B0 M1 A0</p>		

Question Number	Scheme	
6.(a)(i)	$H_0 : \lambda = 7 \quad H_1 : \lambda > 7$  $X = \text{number of visits. } X \sim \text{Po}(7)$  $P(X \geq 10) = 1 - P(X \leq 9) = 0.1695$  $1 - P(X \leq 10) = 0.0985$ $1 - P(X \leq 9) = 0.1695$ $\text{CR } X \geq 11$  $0.1695 > 0.10$ , $\text{CR } X \geq 11$ Not significant or it is not in the critical region or do not reject $H_0$ The rate of visits on a Saturday is not greater/ is unchanged	B1  B1  M1  A1   M1 A1 no ft
(ii)	$X = 11$	B1
(b)	(The visits occur) randomly/ independently or singly or constant rate	B1
(c)	$[H_0 : \lambda = 7 \quad H_1 : \lambda > 7 \quad (\text{or } H_0 : \lambda = 14 \quad H_1 : \lambda > 14)]$  $X \sim N;(14,14)$	
	$P(X \geq 20) = P\left(z \geq \frac{19.5 - 14}{\sqrt{14}}\right)$ $= P(z \geq 1.47)$ $= 0.0708 \quad \text{or } z = 1.2816$	+/- 0.5, stand  M1 M1  A1 dep both M
	$0.0708 < 0.10$ therefore significant. The rate of visits is greater on a Saturday	A1 dep 2 <sup>nd</sup> M
		(6)

Question Number	Scheme	Marks
7. (a)	$F(x_0) = \int_1^x -\frac{2}{9}x + \frac{8}{9} dx = \left[-\frac{1}{9}x^2 + \frac{8}{9}x\right]_1^x$ $= \left[-\frac{1}{9}x^2 + \frac{8}{9}x\right] - \left[-\frac{1}{9} + \frac{8}{9}\right]$ $= -\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9}$	M1A1 A1 (3)
(b)	$F(x) = \begin{cases} 0 & x < 1 \\ -\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} & 1 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$	B1B1√ (2)
(c)	$F(x) = 0.75 ; \quad \text{or } F(2.5) = -\frac{1}{9} \times 2.5^2 + \frac{8}{9} \times 2.5 - \frac{7}{9}$ $-\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} = 0.75$ $4x^2 - 32x + 55 = 0$ $-x^2 + 8x - 13.75 = 0$ $x = 2.5 \quad = 0.75 \quad \text{cso}$ <p>and <math>F(x) = 0.25</math></p> $-\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} = 0.25$ $-x^2 + 8x - 7 = 2.25$ $-x^2 + 8x - 9.25 = 0$ $x = \frac{-8 \pm \sqrt{8^2 - 4 \times -1 \times -9.25}}{2 \times -1}$ $x = 1.40$ <p style="text-align: right;">quadratic 3 terms = 0</p>	M1; A1 M1 M1 dep M1 dep A1 (6)
(d)	$Q_3 - Q_2 > Q_2 - Q_1$ <p>Or mode = 1 and mode &lt; median Or mean = 2 and median &lt; mode Sketch of pdf here or be referred to if in a different part of the question Box plot with <math>Q_1, Q_2, Q_3</math> values marked on Positive skew</p>	M1 A1 (2)