Edexcel S2 – January 2002 – Solutions

1a) A population is a complete collection of items or individuals.

b) A statistic is a random variable calculated as a function of the known observations from a population.

c) The population is the college students, and the statistic is the mean of 75%.

d) The distribution of all possible sample means, for all samples of size 50.

---

2) $H_0 : \lambda = 2.5; \quad H_1 : \lambda > 2.5$

We require a one tailed test of Poisson mean at the 5% significance level.
Assuming Ho true, for 4 weeks, X ~ Po(10)

From tables, critical region is X $\geq$ 16

We are told 14 houses are sold. This is not within the critical region, and so we do not reject Ho. There is insufficient evidence to suggest the new salesman has increased sales.

3a) Let X = "number of passengers who do not show up" then X ~ Bin(200, 0.03)

b) As p is very small, the Poisson approximation may be used. X ~ Po(6)

$P(X < 4) = 0.1512$ (from Cumulative binomial Tables)

c) $P(X > 4) \leq 1 \quad P(X \quad 4) \quad 1 \quad 0.2851 \quad 0.7149$

4a) A possible distribution is the Continuous Uniform Distribution X ~ U[0, 14]

b) By symmetry, Mean = 7

c) Cumulative distribution found by integration.

$$\int_0^x \frac{1}{14}\,dt = \left[\frac{1}{14}t\right]_0^x = \frac{x}{14}$$

$$F(x) = \begin{cases} 0 & x < 0 \\ x/14 & 0 \leq x \leq 14 \\ 1 & x > 14 \end{cases}$$

d) $P(X > 10) = 1 \quad F(10) \quad 1 \quad 10/14 \quad \frac{2}{7}$

5a) Failures occur independently of each other, and randomly within a given time interval at a constant average rate.

bi) Let X = “number of failed attempts in an hour”, then X ~ Po(3)

$P(X = 0) = 0.0498$ (from Cumulative Poisson Tables)

ii) $P(X > 4) \le 1 \quad P(X \quad 4) \quad 1 \quad 0.8153 \quad 0.1847$ (from Cum. Poiss. Tables)

c) Let Y = “number of failed attempts in 8 hours”, then Y ~ Po(24)

d) Using the Normal Approximation, Y ~ N(24, 24)

$$P(Y \ge 12) = P(Y \quad 11.5) \quad P\left(Z \quad \frac{11.5 - 24}{\sqrt{24}}\right)$$

$$- = P(Z \ge \quad 2.55)$$

$$= 0.9946$$

6a) Let X = “number of diners choosing organic food”, then X ~ Bin(20, 0.4)

b) $P(5 < X < 15) \quad P(X \le 14) \quad P(X \quad 5)$ (from tables)

$$= 0.9984 - 0.1256$$

$$= 0.8728$$

c) Mean = 20 x 0.4 = 8; Variance = 8 x 0.6 = 4.8; sd = $\sqrt{4.8} = 2.19$

d) $H_0 : p = 0.4; \quad H_1 : p > 0.4$

We require a one tailed test of Binomial Proportion at the 5% significance level.
Assuming Ho true, X ~ (10, 0.4)

From tables, critical region is X ≥ 8

We are told 8 organic meals are requested. This is within the critical region, so we reject Ho in favour of H_1 concluding there is evidence to suggest the proportion is higher than the trade magazine claims.

7a) As this is a random variable F(0) = 0 and F(2) = 1

$F(2) = 4k + 4k \quad 8k$; as F(2) = 1; $k = 1/8$

b) For median, F(x) = 0.5

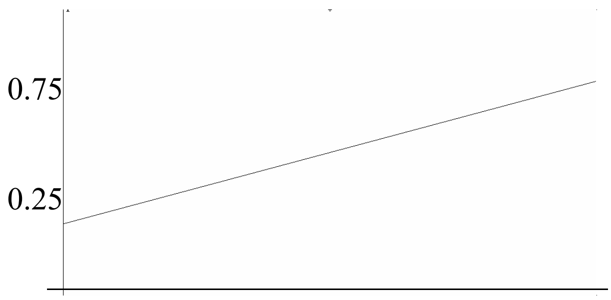
$\frac{1}{8}\left(x^2 + 2x\right) = 0.5$ *i.e.* $x^2 \quad 2x \quad 4 \quad 0$

$$x = 1.236$$

c) f(x) found by differentiation

$$f(x)=\begin{cases}\frac{x}{4}+\frac{1}{4} & 0\le x \quad 2\\ 0 & otherwise\end{cases}$$

d)



e) For mode we require a maximum value.
From graph we can see this occurs when x = 2

f) $$E(X)=\int_0^2 x\left(\frac{1}{4}x+\frac{1}{4}\right)dx \quad \frac{1}{4}\int_0^2 x^2 \quad x\,dx$$

$$=\frac{1}{4}\left[\frac{x^3}{3}+\frac{x^2}{2}\right]_0^2$$

$$=\frac{1}{4}\left(\frac{8}{3}+2\right)$$

$$=\frac{7}{6}$$

g) mean < median < mode hence there is a negative skew.

End of Paper