

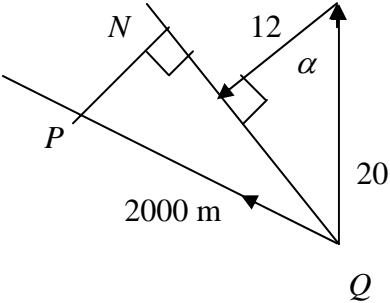
Mark Scheme (Results) Summer 2009

GCE

GCE Mathematics (6680/01)

June 2009
6680 Mechanics M4
Mark Scheme

Question Number	Scheme	Marks
Q1	<p>CLM along plane: $v \cos 30^\circ = u \cos 45^\circ$</p> $v = u \sqrt{\frac{2}{3}}$ <p>Fraction of KE Lost = $\frac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2} = \frac{\frac{1}{2}mu^2 - \frac{1}{2}m\frac{2}{3}u^2}{\frac{1}{2}mu^2} = \frac{1}{3}$</p>	<p>M1 A1 A1 M1 M1 A1 [6]</p>
Q2	$-mg - mkv^2 = ma$ $-(g + kv^2) = v \frac{dv}{dx}$ $\pm \int_0^x dx = \int_{\frac{1}{2}\sqrt{\frac{g}{k}}}^{\frac{1}{2}\sqrt{\frac{g}{k}}} \frac{-v dv}{(g + kv^2)}$ $X = \frac{1}{2k} \left[\ln(g + kv^2) \right]_{\frac{1}{2}\sqrt{\frac{g}{k}}}^{\frac{1}{2}\sqrt{\frac{g}{k}}}$ $= \frac{1}{2k} \left(\ln 2g - \ln \frac{5g}{4} \right)$ $= \frac{1}{2k} \ln \frac{8}{5}$	<p>M1 A1 M1 DM1 A1 (both previous) M1 A1 M1 A1 [9]</p>

Question Number	Scheme	Marks
Q3 (a)	 <p style="text-align: center;">$\cos \alpha = \frac{12}{20}$</p> <p style="text-align: center;">Bearing is $180^\circ + \alpha = 233^\circ$ (nearest degree)</p>	<p>M1</p> <p>M1 A1</p> <p>A1</p> <p style="text-align: right;">(4)</p>
(b)	$PN = 2000\cos(135^\circ - \alpha) = 200\sqrt{2} \text{ m or decimal equivalent}$	<p>M1A1ft A1</p> <p style="text-align: right;">(3)</p>
(c)	$\text{Time to closest approach} = \frac{\sqrt{20^2 - 12^2}}{QN}$ $= \frac{2000\sin(135^\circ - \alpha)}{16}$ <p style="text-align: center;">Distance moved by $Q = \text{time } t \times 12$</p> $= 1050\sqrt{2} \text{ m or decimal equivalent}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>DM1</p> <p>A1</p> <p style="text-align: right;">(5)</p> <p style="text-align: right;">[12]</p>

Question Number	Scheme	Marks
Q4 (a)	$V = -mg2a \sin 2\theta - \frac{7}{20}mg(L - 4a \sin \theta)$ $= \frac{1}{5}mga(7 \sin \theta - 10 \sin 2\theta) - \frac{7}{20}mgL$	M1 B1 A1 A1 (4)
(b)	$\frac{dV}{d\theta} = \frac{1}{5}mga(7 \cos \theta - 20 \cos 2\theta)$ $\frac{1}{5}mga(7 \cos \theta - 20 \cos 2\theta) = 0$ $7 \cos \theta - 20(2 \cos^2 \theta - 1) = 0$ $40 \cos^2 \theta - 7 \cos \theta - 20 = 0$ $(5 \cos \theta - 4)(8 \cos \theta + 5) = 0$ $\cos \theta = \frac{4}{5} \text{ or } (\cos \theta = -\frac{5}{8} \Rightarrow 2\theta > 180^\circ)$	M1 A1 DM1 DM1 A1 DM1 A1 DM1 (8)
(c)	$\frac{d^2V}{d\theta^2} = \frac{1}{5}mga(-7 \sin \theta + 40 \sin 2\theta)$ $= \frac{1}{5}mga(-7 \sin \theta + 80 \sin \theta \cos \theta)$ <p>When $\cos \theta = \frac{4}{5}$,</p> $\frac{d^2V}{d\theta^2} = \frac{1}{5}mga\left(-\frac{21}{5} + 80 \times \frac{3}{5} \times \frac{4}{5}\right) = \frac{171}{25}mga$ <p>> 0 therefore stable</p>	M1 A1 M1 A1 cso (4) [16]

Question Number	Scheme	Marks
Q5 (a)	<p>CLM: $2(\mathbf{i} + 2\mathbf{j}) + -2\mathbf{i} = 2\mathbf{j} + \mathbf{v}$ $\mathbf{v} = 2\mathbf{j} \text{ m s}^{-1}$</p>	<p>M1 A1 A1 (3)</p>
(b)	<p>$\mathbf{I} = 2(\mathbf{j} - (\mathbf{i} + 2\mathbf{j}))$ $= (-2\mathbf{i} - 2\mathbf{j}) \text{ Ns}$ Since \mathbf{I} acts along l.o.c.c. , l.o.c.c is parallel to $\mathbf{i} + \mathbf{j}$</p>	<p>M1 A1 A1 B1 (4)</p>
(c)	<p>Before A: $(\mathbf{i} + 2\mathbf{j}) \cdot \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \frac{3}{\sqrt{2}}$ B: $-2\mathbf{j} \cdot \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \frac{-2}{\sqrt{2}}$</p> <p>After A: $\mathbf{j} \cdot \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \frac{1}{\sqrt{2}}$ B: $2\mathbf{j} \cdot \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \frac{2}{\sqrt{2}}$</p> <p>NIL:</p> $e = \frac{\frac{2}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{\frac{3}{\sqrt{2}} - \frac{-2}{\sqrt{2}}} = \frac{1}{5}$	<p>M1 A3</p>
		<p>DM1 A1 (6) [13]</p>

Question Number	Scheme	Marks
Q6 (a)	$(\rightarrow), T = m\ddot{y}$ <p>Hooke's Law:</p> $T = \frac{2mn^2ax}{2a} = mn^2x$ $\left. \begin{aligned} x + y &= \frac{1}{2}ft^2 \\ \dot{x} + \dot{y} &= ft \\ \ddot{x} + \ddot{y} &= f \end{aligned} \right\}$ <p>so, $(\rightarrow), mn^2x = m\ddot{y} = m(f - \ddot{x})$</p> $\ddot{x} + n^2x = f^{**}$	<p>M1</p> <p>B1</p> <p>B2</p> <p>DM1</p> <p>A1</p>
(b)	<p>C.F. : $x = A \cos nt + B \sin nt$</p> <p>P.I. : $x = \frac{f}{n^2}$</p> <p>Gen solution: $x = A \cos nt + B \sin nt + \frac{f}{n^2}$</p> $\dot{x} = -An \sin nt + Bn \cos nt$ $\left. \begin{aligned} t = 0, x = 0 &\Rightarrow A = -\frac{f}{n^2} \\ t = 0, \dot{x} = 0 &\Rightarrow B = 0 \end{aligned} \right\}$ $x = \frac{f}{n^2}(1 - \cos nt)$	<p>(6)</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>M1 A1ft</p> <p>M1 A1</p> <p>A1</p>
(c)	$\dot{x} = 0 \Rightarrow nt = \pi$ $x_{\max} = \frac{f}{n^2}(1 - (-1)) = \frac{2f}{n^2}$	<p>(8)</p> <p>M1</p> <p>M1 A1</p>
(d)	$\dot{y} = ft - \dot{x}$ $= f \frac{\pi}{n} - 0 = \frac{f\pi}{n}$	<p>(3)</p> <p>M1</p> <p>A1</p> <p>(2)</p> <p>[19]</p>