

1. A particle P moves along the x -axis. At time $t = 0$, P passes through the origin O , moving in the positive x -direction. At time t seconds, the velocity of P is $v \text{ m s}^{-1}$ and $OP = x$ metres. The acceleration of P is $\frac{1}{12}(30 - x) \text{ m s}^{-2}$, measured in the positive x -direction.

(a) Give a reason why the maximum speed of P occurs when $x = 30$.

(1)

Given that the maximum speed of P is 10 m s^{-1} ,

(b) find an expression for v^2 in terms of x .

(5)



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Question 1 continued

Handwriting practice area consisting of 30 horizontal lines.

(Total 6 marks)

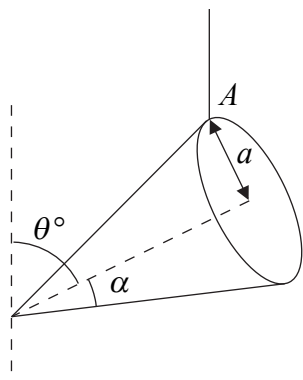
Q1



N 2 3 8 2 2 A 0 3 2 4

2.

Figure 1



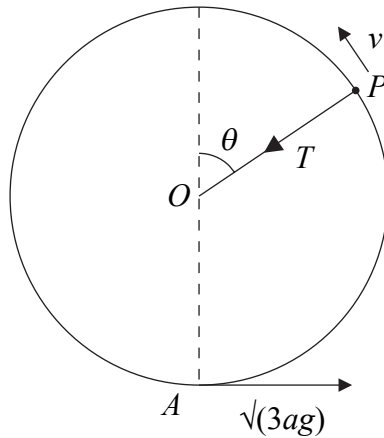
A uniform solid right circular cone has base radius a and semi-vertical angle α , where $\tan \alpha = \frac{1}{3}$. The cone is freely suspended by a string attached at a point A on the rim of its base, and hangs in equilibrium with its axis of symmetry making an angle of θ° with the upward vertical, as shown in Figure 1.

Find, to one decimal place, the value of θ .



4.

Figure 2



A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a point O . The point A is vertically below O , and $OA = a$. The particle is projected horizontally from A with speed $\sqrt{3ag}$. When OP makes an angle θ with the upward vertical through O and the string is still taut, the tension in the string is T and the speed of P is v , as shown in Figure 2.

(a) Find, in terms of a , g and θ , an expression for v^2 . (3)

(b) Show that $T = (1 - 3 \cos \theta)mg$. (3)

The string becomes slack when P is at the point B .

(c) Find, in terms of a , the vertical height of B above A . (2)

After the string becomes slack, the highest point reached by P is C .

(d) Find, in terms of a , the vertical height of C above B . (5)



Question 4 continued

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Lined area for writing answer to Question 4.

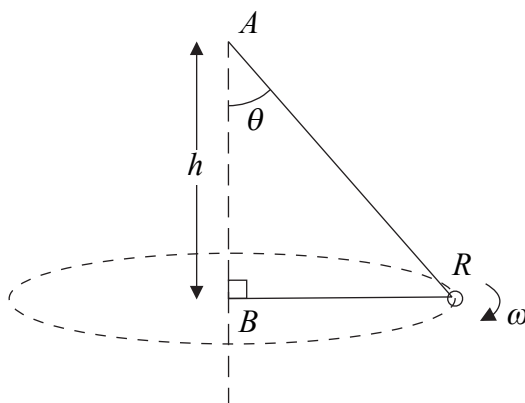
(Total 13 marks)

Q4



5.

Figure 3



One end of a light inextensible string is attached to a fixed point A . The other end of the string is attached to a fixed point B , vertically below A , where $AB = h$. A small smooth ring R of mass m is threaded on the string. The ring R moves in a horizontal circle with centre B , as shown in Figure 3. The upper section of the string makes a constant angle θ with the downward vertical and R moves with constant angular speed ω . The ring is modelled as a particle.

(a) Show that $\omega^2 = \frac{g}{h} \left(\frac{1 + \sin \theta}{\sin \theta} \right)$. (7)

(b) Deduce that $\omega > \sqrt{\frac{2g}{h}}$. (2)

Given that $\omega = \sqrt{\frac{3g}{h}}$,

(c) find, in terms of m and g , the tension in the string. (4)



Question 5 continued

Lined writing area for the answer to Question 5.

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(Total 13 marks)

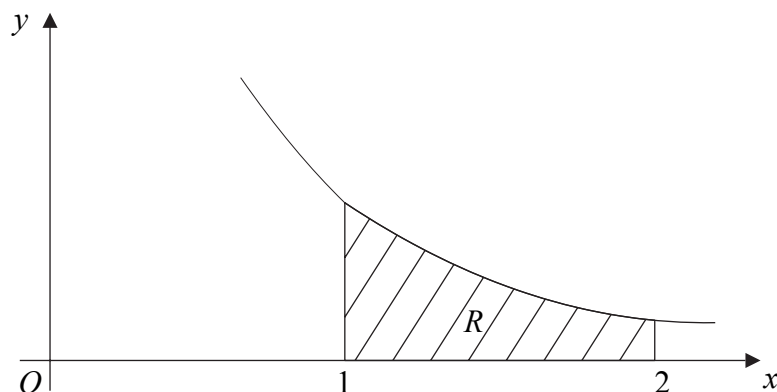
Q5

Grade boxes for Question 5.



6.

Figure 4

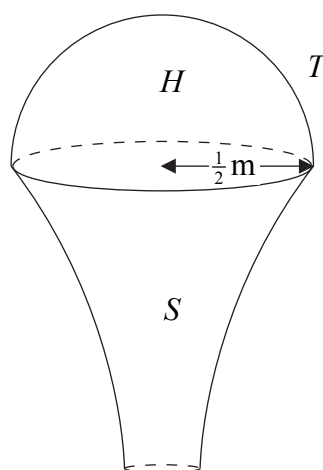


The shaded region R is bounded by the curve with equation $y = \frac{1}{2x^2}$, the x -axis and the lines $x = 1$ and $x = 2$, as shown in Figure 4. The unit of length on each axis is 1 m. A uniform solid S has the shape made by rotating R through 360° about the x -axis.

(a) Show that the centre of mass of S is $\frac{2}{7}$ m from its larger plane face.

(6)

Figure 5



A sporting trophy T is a uniform solid hemisphere H joined to the solid S . The hemisphere has radius $\frac{1}{2}$ m and its plane face coincides with the larger plane face of S , as shown in Figure 5. Both H and S are made of the same material.

(b) Find the distance of the centre of mass of T from its plane face.

(7)



7. A particle P of mass 0.25 kg is attached to one end of a light elastic string. The string has natural length 0.8 m and modulus of elasticity λ N. The other end of the string is attached to a fixed point A . In its equilibrium position, P is 0.85 m vertically below A .

(a) Show that $\lambda = 39.2$. (2)

The particle is now displaced to a point B , 0.95 m vertically below A , and released from rest.

(b) Prove that, while the string remains stretched, P moves with simple harmonic motion of period $\frac{\pi}{7}$ s. (6)

(c) Calculate the speed of P at the instant when the string first becomes slack. (3)

The particle first comes to instantaneous rest at the point C .

(d) Find, to 3 significant figures, the time taken for P to move from B to C . (5)



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