

# Mark Scheme (Final)

## January 2009

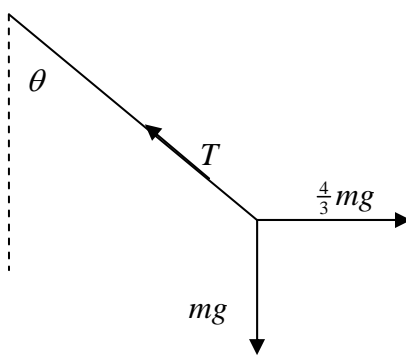
GCE

### GCE Mechanics M3 (6679/01)

## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

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6679 Mechanics M3  
Mark Scheme

Question Number	Scheme	Marks
1.	<p>N2L</p> $3a = -\left(9 + \frac{15}{(t+1)^2}\right)$ $3v = -9t + \frac{15}{t+1} (+A)$ $v=0, t=4 \Rightarrow 0 = -36 + 3 + A \Rightarrow A = 33$ $v = -3t + \frac{5}{t+1} + 11$ $t=0 \Rightarrow v=16$	<p>B1</p> <p>M1 A1ft</p> <p>M1 A1</p> <p>M1 A1 (7) [7]</p>
2.	<div style="text-align: center;">  </div> <p>(a) (<math>\leftarrow</math>) <math>T \sin \theta = \frac{4}{3}mg</math></p> <p>(<math>\uparrow</math>) <math>T \cos \theta = mg</math></p> $T^2 = \left(\frac{4}{3}mg\right)^2 + (mg)^2$ <p>Leading to <math>T = \frac{5}{3}mg</math></p> <p>(b) HL <math>T = \frac{\lambda x}{a} \Rightarrow \frac{5}{3}mg = \frac{3mge}{a}</math></p> $e = \frac{5}{9}a$ $E = \frac{\lambda x^2}{2a} = \frac{3mg}{2a} \times \left(\frac{5}{9}a\right)^2 = \frac{25}{54}mga$	<p>M1 A1</p> <p>A1</p> <p>M1</p> <p>A1 (5)</p> <p>M1 A1ft ft their T</p> <p>M1 A1 (4) [9]</p>

Question Number	Scheme	Marks
3.	$\omega = \frac{80 \times 2\pi}{60} \text{ rad s}^{-1} \left( = \frac{8\pi}{3} \approx 8.377... \right)$ <p style="text-align: center;">Accept <math>v = \frac{16\pi}{75} \approx 0.67 \text{ ms}^{-1}</math> as equivalent</p> $(\uparrow) R = mg$ <p>For least value of <math>\mu</math> <math>(\leftarrow) \mu mg = mr\omega^2</math></p> $\mu = \frac{0.08}{9.8} \times \left( \frac{8\pi}{3} \right)^2 \approx 0.57$ <p style="text-align: right;">accept 0.573</p>	<p>B1</p> <p>B1</p> <p>M1 A1=A1</p> <p>M1 A1 (7)</p> <p>[7]</p>
4.	<p>(a)</p> $a = 8$ $T = \frac{25}{2} = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{4\pi}{25} (\approx 0.502 \dots)$ $v^2 = \omega^2 (a^2 - x^2) \Rightarrow v^2 = \left( \frac{4\pi}{25} \right)^2 (8^2 - 3^2)$ $v = \frac{4\pi}{25} \sqrt{55} \approx 3.7 \text{ (mh}^{-1}\text{)}$ <p>(b)</p> $x = a \cos \omega t \Rightarrow 3 = 8 \cos \left( \frac{4\pi}{25} t \right)$ $t \approx 2.3602 \dots$ <p>time is 12 22</p>	<p>B1</p> <p>M1 A1</p> <p>M1 A1ft</p> <p>M1 A1 (7)</p> <p>M1 A1ft</p> <p>M1 A1 (4)</p> <p>[11]</p>

Question Number	Scheme	Marks
5.	<p>(a) Let <math>x</math> be the distance from the initial position of <math>B</math> to <math>C</math>  GPE lost = EPE gained  <math display="block">mgx \sin 30^\circ = \frac{6mgx^2}{2a}</math> Leading to <math>x = \frac{a}{6}</math>  <math display="block">AC = \frac{7a}{6}</math></p> <p>(b) The greatest speed is attained when the acceleration of <math>B</math> is zero, that is where the forces on <math>B</math> are equal.  (↖) <math display="block">T = mg \sin 30^\circ = \frac{6mge}{a}</math> <math display="block">e = \frac{a}{12}</math> CE <math display="block">\frac{1}{2}mv^2 + \frac{6mg}{2a} \left(\frac{a}{12}\right)^2 = mg \frac{a}{12} \sin 30^\circ</math> Leading to <math display="block">v = \sqrt{\left(\frac{ga}{24}\right)} = \frac{\sqrt{6ga}}{12}</math></p> <p><i>Alternative approaches to (b) are considered on the next page.</i></p>	<p>M1 A1=A1  M1  A1 (5)</p> <p>M1  A1  M1 A1=A1  M1 A1 (7)  [12]</p>

Question Number	Scheme	Marks
5.	<p><i>Alternative approach to (b) using calculus with energy.</i></p> <p>Let distance moved by <math>B</math> be <math>x</math></p> <p>CE <math display="block">\frac{1}{2}mv^2 + \frac{6mg}{2a}x^2 = mgx \sin 30^\circ</math></p> $v^2 = gx - \frac{6g}{a}x^2$ <p>For maximum <math>v</math> <math display="block">\frac{d}{dx}(v^2) = 2v \frac{dv}{dx} = g - \frac{12g}{a}x = 0</math></p> $x = \frac{a}{12}$ $v^2 = g\left(\frac{a}{12}\right) - \frac{6g}{a}\left(\frac{a}{12}\right)^2 = \frac{ga}{24}$ $v = \sqrt{\left(\frac{ga}{24}\right)}$	<p>M1 A1=A1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (7)</p>
	<p><i>Alternative approach to (b) using calculus with Newton's second law.</i></p> <p>As before, the centre of the oscillation is when extension is <math>\frac{a}{12}</math></p> <p>N2L <math display="block">mg \sin 30^\circ - T = m\ddot{x}</math></p> $\frac{1}{2}mg - \frac{6mg\left(\frac{a}{12} + x\right)}{a} = m\ddot{x}$ $\ddot{x} = -\frac{6g}{a}x \Rightarrow \omega^2 = \frac{6g}{a}$ $v_{\max} = \omega a = \sqrt{\left(\frac{6g}{a}\right)} \times \frac{a}{12} = \sqrt{\left(\frac{ga}{24}\right)}$	<p>M1 A1</p> <p>M1 A1</p> <p>A1</p> <p>M1 A1 (7)</p>

Question Number	Scheme	Marks
6.	<p>(a)</p> $\int y^2 dx = \int (4-x^2)^2 dx = \int (16-8x^2+x^4) dx$ $= 16x - \frac{8x^3}{3} + \frac{x^5}{5}$ $\left[ 16x - \frac{8x^3}{3} + \frac{x^5}{5} \right]_0^2 = \frac{256}{15}$ $\int xy^2 dx = \int x(4-x^2)^2 dx = \int (16x-8x^3+x^5) dx$ $= 8x^2 - 2x^4 + \frac{x^6}{6}$ $\left[ 8x^2 - 2x^4 + \frac{x^6}{6} \right]_0^2 = \frac{32}{3}$ $\bar{x} = \frac{\int xy^2 dx}{\int y^2 dx} = \frac{32}{3} \times \frac{15}{216} = \frac{5}{8} *$ <p>(b)</p> $A \times \bar{x} = (\pi r^2 l) \times \frac{l}{2}$ $\frac{256}{15} \pi \times \frac{5}{8} = \pi \times 16l \times \frac{l}{2}$ <p>Leading to <math>l = \frac{2\sqrt{3}}{3}</math> accept exact equivalents or awrt 1.15</p>	<p>M1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1A1</p> <p>M1 A1 (10)</p> <p>M1</p> <p>A1 ft</p> <p>M1 A1 (4)</p> <p>[14]</p>

Question Number	Scheme	Marks
7.	<p>(a) Let speed at C be <math>u</math></p> <p>CE <math>\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{ag}{4}\right) = mga(1 - \cos\theta)</math></p> $u^2 = \frac{9ga}{4} - 2ga \cos\theta$ <p>(□) <math>mg \cos\theta (+R) = \frac{mu^2}{a}</math></p> $mg \cos\theta = \frac{9mg}{4} - 2mg \cos\theta \quad \text{eliminating } u$ <p>Leading to <math>\cos\theta = \frac{3}{4} *</math></p> <p>(b) At C <math>u^2 = \frac{9ga}{4} - 2ga \times \frac{3}{4} = \frac{3}{4}ga</math></p> <p>(→) <math>u_x = u \cos\theta = \sqrt{\left(\frac{3ga}{4}\right)} \times \frac{3}{4} = \sqrt{\left(\frac{27ga}{64}\right)} = 2.033\sqrt{a}</math></p> <p>(↓) <math>u_y = u \sin\theta = \sqrt{\left(\frac{3ga}{4}\right)} \times \frac{\sqrt{7}}{4} = \sqrt{\left(\frac{21ga}{64}\right)} = 1.792\sqrt{a}</math></p> $v_y^2 = u_y^2 + 2gh \Rightarrow v_y^2 = \frac{21}{64}ga + 2g \times \frac{7}{4}a = \frac{245}{64}ga$ $\tan\psi = \frac{v_y}{u_x} = \sqrt{\left(\frac{245}{27}\right)} \approx 3.012 \dots$ <p><math>\psi \approx 72^\circ</math> awrt <math>72^\circ</math></p> <p>Or <math>1.3^\circ</math> (1.2502°) awrt <math>1.3^\circ</math></p>	<p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>M1 A1 (7)</p> <p>B1</p> <p>M1 A1ft</p> <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (8)</p> <p>[15]</p>
	<p><i>Alternative for the last five marks</i></p> <p>Let speed at P be <math>v</math>.</p> <p>CE <math>\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{ag}{4}\right) = mg \times 2a</math> or equivalent</p> $v^2 = \frac{17mga}{4}$ $\cos\psi = \frac{u_x}{v} = \sqrt{\left(\frac{27}{64} \times \frac{4}{17}\right)} = \sqrt{\left(\frac{27}{272}\right)} \approx 0.315$ <p><math>\psi \approx 72^\circ</math> awrt <math>72^\circ</math></p> <p><i>Note: The time of flight from C to P is <math>\frac{\sqrt{235} - \sqrt{21}}{8} \sqrt{\left(\frac{a}{g}\right)} \approx 1.38373 \sqrt{\left(\frac{a}{g}\right)}</math></i></p>	<p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p>