

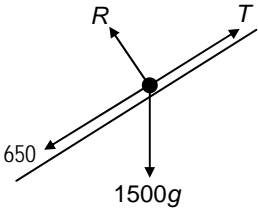
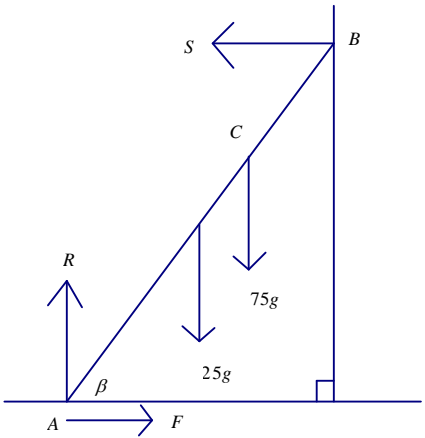
Mark Scheme (Results)

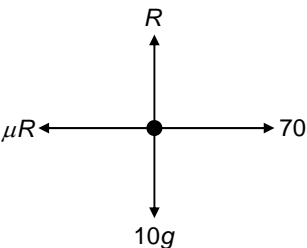
January 2009






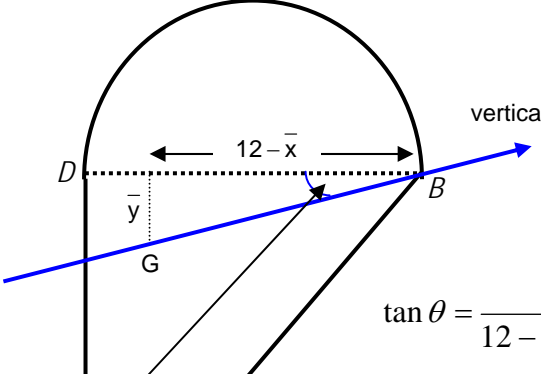
GCE

GCE Mathematics (6678/01)

January 2009
6678 Mechanics M2
Mark Scheme

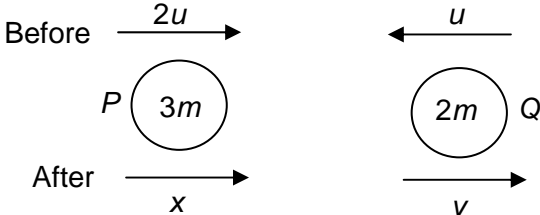
Question Number	Scheme	Marks
1	 <p style="margin-left: 40px;"> $F = ma$ parallel to the slope, $T - 1500g \sin \theta - 650 = 1500a$ Tractive force, $30000 = T \times 15$ $a = \frac{\frac{30000}{15} - 1500(9.8)(\frac{1}{14}) - 650}{1500}$ $\underline{0.2} \text{ (m s}^{-2}\text{)}$ </p>	M1* A1 M1* d*M1 A1 (5) [5]
2	<div style="display: flex; flex-direction: column;"> <div style="margin-bottom: 20px;"> <p>(a)</p>  </div> <div> <p>(b)</p> <p style="margin-left: 40px;"> $R(\uparrow): R = 25g + 75g (= 100g)$ $F = \mu R \Rightarrow F = \frac{11}{25} \times 100g$ $= 44g (= 431)$ </p> <p style="margin-left: 40px;"> $M(A):$ $25g \times 2 \cos \beta + 75g \times 2.8 \cos \beta$ $= S \times 4 \sin \beta$ </p> <p style="margin-left: 40px;"> $R(\leftrightarrow): F = S$ $176g \sin \beta = 260g \cos \beta$ $\beta = 56^\circ$ </p> </div> <div> <p>(c) So that Reece's weight acts directly at the point C.</p> </div> </div>	B1 M1 A1 (3) M1 A2,1,0 M1A1 A1 (6) B1 [10]

Question Number	Scheme	Marks
<p>3 (a)</p> 	<p>$R(\uparrow) : R = 10g$</p> <p>$F = \mu R \Rightarrow F = \frac{4}{7}(10g) = 56$</p> <p>$\therefore$ WD against friction = $\frac{4}{7}(10g)(50)$</p> <p>2800(J)</p> <p>(b) $70(50) - "2800" = \frac{1}{2}(10)v^2 - \frac{1}{2}(10)(2)^2$</p> <p>$700 = 5v^2 - 20, 5v^2 = 720 \Rightarrow v^2 = 144$</p> <p>Hence, $v = \underline{12}$ (m s⁻¹)</p> <p>Or (b) N2L(\rightarrow): $70 - \frac{4}{7}R = 10a$</p> <p>$70 - \frac{4}{7} \times 10g = 10a, (a = 1.4)$</p> <p>AB($\rightarrow$): $v^2 = (2)^2 + 2(1.4)(50)$</p> <p>Hence, $v = \underline{12}$ (m s⁻¹)</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>(4)</p> <p>M1*</p> <p>A1ft</p> <p>d*M1</p> <p>A1 cao</p> <p>(4)</p> <p>M1*</p> <p>A1ft</p> <p>d*M1</p> <p>A1 cao</p> <p>(4)</p> <p>[8]</p>
<p>4 (a)</p>	<p>$v = 10t - 2t^2, s = \int v dt$</p> <p>$= 5t^2 - \frac{2t^3}{3} (+C)$</p> <p>$t = 6 \Rightarrow s = 180 - 144 = \underline{36}$ (m)</p> <p>(b) $\underline{s} = \int v dt = \frac{-432t^{-1}}{-1} (+K) = \frac{432}{t} (+K)$</p> <p>$t = 6, s = "36" \Rightarrow 36 = \frac{432}{6} + K$</p> <p>$\Rightarrow K = -36$</p> <p>At $t = 10, s = \frac{432}{10} - 36 = \underline{7.2}$ (m)</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>(3)</p> <p>B1</p> <p>M1*</p> <p>A1</p> <p>d*M1</p> <p>A1</p> <p>(5)</p> <p>[8]</p>

Question Number	Scheme			Marks	
5 (a)	MR	 108	 18π	 $108 + 18\pi$	B1
	x_i (\rightarrow) from AD	4	6	\bar{x}	B1
	y_i (\downarrow) from BD	6	$-\frac{8}{\pi}$	\bar{y}	
	$AD(\rightarrow): 108(4) + 18\pi(6) = (108 + 18\pi)\bar{x}$				M1
	$\bar{x} = \frac{432 + 108\pi}{108 + 18\pi} = 4.68731\dots = \underline{4.69}$ (cm) (3 sf) AG				A1 (4)
(b)	y_i (\downarrow) from BD	 6	 $-\frac{8}{\pi}$	\bar{y}	B1 oe
	$BD(\downarrow): 108(6) + 18\pi(-\frac{8}{\pi}) = (108 + 18\pi)\bar{y}$				M1 A1ft
	$\bar{y} = \frac{504}{108 + 18\pi} = 3.06292\dots = \underline{3.06}$ (cm) (3 sf)				A1 (4)
(c)	 <p style="text-align: right;"> $\tan \theta = \frac{\bar{y}}{12 - 4.68731\dots}$ $= \frac{3.06392\dots}{12 - 4.68731\dots}$ </p>				M1 dM1 A1
	$\theta = 22.72641\dots = \underline{23}$ (nearest degree)				A1 (4) [12]

Question Number	Scheme	Marks
6	<p>(a) Horizontal distance: $57.6 = p \times 3$ $p = 19.2$</p> <p>(b) Use $s = ut + \frac{1}{2}at^2$ for vertical displacement. $-0.9 = q \times 3 - \frac{1}{2}g \times 3^2$ $-0.9 = 3q - \frac{9g}{2} = 3q - 44.1$ $q = \frac{43.2}{3} = 14.4$ *AG*</p> <p>(c) initial speed $\sqrt{p^2 + 14.4^2}$ (with their p) $= \sqrt{576} = \underline{24}$ (m s⁻¹)</p> <p>(d) $\tan \alpha = \frac{14.4}{p} (= \frac{3}{4})$ (with their p)</p> <p>(e) When the ball is 4 m above ground: $3.1 = ut + \frac{1}{2}at^2$ used $3.1 = 14.4t - \frac{1}{2}gt^2$ o.e. ($4.9t^2 - 14.4t + 3.1 = 0$) $\Rightarrow t = \frac{14.4 \pm \sqrt{(14.4)^2 - 4(4.9)(3.1)}}{2(4.9)}$ seen or implied $t = \frac{14.4 \pm \sqrt{146.6}}{9.8} = 0.023389... \text{ or } 2.70488...$ awrt 0.23 and 2.7 duration = $2.70488... - 0.023389...$ $= 2.47$ or 2.5 (seconds)</p>	<p>M1 A1 (2)</p> <p>M1 A1 A1 cso (3)</p> <p>M1 A1 cao (2)</p> <p>B1 (1)</p> <p>M1 A1 M1 A1 (6)</p>
or 6	<p>(e) M1A1M1 as above $t = \frac{14.4 \pm \sqrt{146.6}}{9.8}$ Duration $2 \times \frac{\sqrt{146.6}}{9.8}$ o.e. $= 2.47$ or 2.5 (seconds)</p> <p>(f) Eg. : Variable 'g', Air resistance, Speed of wind, Swing of ball, The ball is not a particle.</p>	<p>A1 M1 A1 (6)</p> <p>B1 (1)</p>

[15]

Question Number	Scheme	Marks
7	<p>(a) </p> <p>Correct use of NEL</p> <p>$y - x = e(2u + u)$ o.e.</p> <p>CLM (\rightarrow): $3m(2u) + 2m(-u) = 3m(x) + 2m(y)$ ($\Rightarrow 4u = 3x + 2y$)</p> <p>Hence $x = y - 3eu$, $4u = 3(y - 3eu) + 2y$, ($u(9e + 4) = 5y$)</p> <p>Hence, speed of $Q = \frac{1}{5}(9e + 4)u$ AG</p> <p>(b) $x = y - 3eu = \frac{1}{5}(9e + 4)u - 3eu$</p> <p>Hence, speed P $= \frac{1}{5}(4 - 6e)u = \frac{2u}{5}(2 - 3e)$ o.e.</p> <p>$x = \frac{1}{2}u = \frac{2u}{5}(2 - 3e) \Rightarrow 5u = 8u - 12eu, \Rightarrow 12e = 3$ & solve for e</p> <p>gives, $e = \frac{3}{12} \Rightarrow e = \frac{1}{4}$ AG</p> <p>Or (b) Using NEL correctly with given speeds of P and Q</p> <p>$3eu = \frac{1}{5}(9e + 4)u - \frac{1}{2}u$</p> <p>$3eu = \frac{9}{5}eu + \frac{4}{5}u - \frac{1}{2}u$, $3e - \frac{9}{5}e = \frac{4}{5} - \frac{1}{2}$ & solve for e</p> <p>$\frac{6}{5}e = \frac{3}{10} \Rightarrow e = \frac{15}{60} \Rightarrow e = \frac{1}{4}$.</p> <p>(c) Time taken by Q from A to the wall $= \frac{d}{y} = \left\{ \frac{4d}{5u} \right\}$</p> <p>Distance moved by P in this time $= \frac{u}{2} \times \frac{d}{y} = \left(= \frac{u}{2} \left(\frac{4d}{5u} \right) = \frac{2}{5}d \right)$</p> <p>Distance of P from wall $= d - x \left(\frac{d}{y} \right); = d - \frac{2}{5}d = \frac{3}{5}d$ AG</p> <p>or (c) Ratio speed P: speed Q $= x:y = \frac{1}{2}u : \frac{1}{5} \left(\frac{9}{4} + 4 \right)u = \frac{1}{2}u : \frac{5}{4}u = 2:5$</p> <p>So if Q moves a distance d, P will move a distance $\frac{2}{5}d$</p> <p>Distance of P from wall $= d - \frac{2}{5}d; = \frac{3}{5}d$ AG</p>	<p>M1*</p> <p>A1</p> <p>B1</p> <p>d*M1</p> <p>A1 cso</p> <p>(5)</p> <p>M1#</p> <p>A1</p> <p>d#M1</p> <p>A1</p> <p>(4)</p> <p>M1#</p> <p>A1</p> <p>d#M1</p> <p>A1</p> <p>(4)</p> <p>M1†</p> <p>A1</p> <p>d†M1;</p> <p>A1 cso</p> <p>(4)</p> <p>M1†</p> <p>A1</p> <p>d†M1; A1</p> <p>(4)</p> <p>cso</p>

Question Number	Scheme	Marks
(d)	<p>After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y</p> <p>Time for P, $T_{AB} = \frac{\frac{3d}{5} - x}{\frac{1}{2}u}$, Time for Q, $T_{WB} = \frac{x}{\frac{1}{4}u}$ from their y</p> <p>Hence $T_{AB} = T_{WB} \Rightarrow \frac{\frac{3d}{5} - x}{\frac{1}{2}u} = \frac{x}{\frac{1}{4}u}$</p> <p>gives, $2\left(\frac{3d}{5} - x\right) = 4x \Rightarrow \frac{3d}{5} - x = 2x, 3x = \frac{3d}{5} \Rightarrow x = \frac{1}{5}d$</p>	<p>B1ft</p> <p>B1ft</p> <p>M1</p> <p>A1 cao</p> <p>(4)</p>
or (d)	<p>After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y</p> <p>speed $P = x = \frac{1}{2}u$, speed P: new speed $Q = \frac{1}{2}u : \frac{1}{4}u = 2:1$ from their y</p> <p>Distance of B from wall = $\frac{1}{3} \times \frac{3d}{5} = \frac{d}{5}$ their $\frac{1}{2+1}$</p>	<p>B1ft</p> <p>B1ft</p> <p>M1; A1</p> <p>(4)</p>
2 nd or (d)	<p>After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y</p> <p>Combined speed of P and Q = $\frac{1}{2}u + \frac{1}{4}u = \frac{3}{4}u$</p> <p>Time from wall to 2nd collision = $\frac{\frac{3d}{5}}{\frac{3u}{4}} = \frac{3d}{5} \times \frac{4}{3u} = \frac{4d}{5u}$ from their y</p> <p>Distance of B from wall = (their speed)x(their time) = $\frac{u}{4} \times \frac{4d}{5u} = \frac{1}{5}d$</p>	<p>B1ft</p> <p>B1ft</p> <p>M1; A1</p> <p>(4)</p> <p>[17]</p>