GCE Examinations

Further Pure Mathematics Module FP3

Advanced Subsidiary / Advanced Level

Paper G

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 8 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Rosemary Smith & Shaun Armstrong

1. $$\mathbf{A} = \begin{pmatrix} 3 & 1 & -4 \\ 1 & 2 & -1 \\ 2 & k & 0 \end{pmatrix}.$$

Find the value of the constant k for which **A** is a singular matrix. **(3 marks)**

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2. Solve the equation

$$z^3 = -4 + 4\sqrt{3}\,\mathrm{i},$$

giving your answers in the form $r(\cos\theta + \mathrm{i}\sin\theta)$ where $r > 0$ and $0 \leq \theta < 2\pi$. **(6 marks)**

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3. Prove by induction that $n(n^2 + 5)$ is divisible by 6 for all positive integers n. **(7 marks)**

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4. The point P represents the complex number z in an Argand diagram.

Given that

$$|\,z - 1 + 2\mathrm{i}\,| = 3,$$

(a) sketch the locus of P in an Argand diagram. **(3 marks)**

T, U and V are transformations from the z-plane to the w-plane where

$$T: \quad w = 4z,$$

$$U: \quad w = z + 5 - \mathrm{i},$$

$$V: \quad w = z\mathrm{e}^{\mathrm{i}\frac{\pi}{2}}.$$

(b) Describe exactly the locus of the image of P under each of these transformations.

(6 marks)

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5. *(a)* By finding the first four derivatives of $f(x) = \cos x$, find the Taylor series expansion of $f(x)$ in ascending powers of $\left(x - \frac{\pi}{6}\right)$ up to and including the term in $\left(x - \frac{\pi}{6}\right)^3$.

(5 marks)

(b) Use this expansion to find an estimate of $\cos \frac{\pi}{4}$, giving your answer to 4 decimal places.

(3 marks)

(c) Find the percentage error in your answer to part *(b)*, giving your answer to 2 significant figures.

(2 marks)

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6. Given that y satisfies the differential equation

$$\frac{d^2y}{dx^2} = x^2 + xy - y^2, \quad y = \tfrac{1}{2} \text{ and } \frac{dy}{dx} = -1 \text{ at } x = 0,$$

(a) use the Taylor series method to obtain a series for y in ascending powers of x up to and including the term in x^3.

(6 marks)

(b) Use your series to estimate the value of y at $x = -0.1$ **(1 mark)**

(c) Use the approximation $\left(\frac{d^2y}{dx^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2}$ with a step length of 0.1 and your answer to part *(b)* to estimate the value of y when $x = 0.1$

(3 marks)

Turn over

7. Referred to a fixed origin, the straight lines l_1, l_2 and l_3 have equations

$$l_1: \ \mathbf{r} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k} + s(2\mathbf{i} - 4\mathbf{j} + \mathbf{k}),$$
$$l_2: \ \mathbf{r} = 3\mathbf{i} + 4\mathbf{k} + t(4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}),$$
$$l_3: \ \mathbf{r} = \mathbf{i} - 2\mathbf{j} + u(2\mathbf{j} + \mathbf{k}).$$

The acute angle between l_1 and l_2 is θ.

(a) Find the exact value of $\sin\theta$. **(5 marks)**

The plane Π contains the lines l_1 and l_2.

(b) Find an equation of Π, giving your answer in the form $ax + by + cz + d = 0$. **(4 marks)**

(c) Show that the line l_3 lies on the plane Π. **(4 marks)**

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8. *(a)* **A** and **B** are non-singular square matrices. Prove that $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$. **(4 marks)**

The transformations $S : \mathbb{R}^2 \to \mathbb{R}^2$ and $T : \mathbb{R}^2 \to \mathbb{R}^2$ are defined by

$$S : \begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} y - x \\ 2x + y \end{pmatrix} \quad \text{and} \quad T : \begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 3x \\ x + y \end{pmatrix}.$$

(b) Show that S represents a linear transformation. **(7 marks)**

(c) Using your result in *(a)*, or otherwise, find the matrix that represents the transformation $(ST)^{-1}$.

(6 marks)

END