

GCE Examinations

Further Pure Mathematics

Module FP3

Advanced Subsidiary / Advanced Level

Paper F

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and/or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working will gain no credit.



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1. Prove by induction that, for all $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^n \ln \frac{r+1}{r} = \ln(n+1). \quad \text{(6 marks)}$$

2.
$$\mathbf{M} = \begin{pmatrix} 2 & 3 \\ 3 & -6 \end{pmatrix}.$$

(a) Find the eigenvalues of \mathbf{M} . (4 marks)

(b) Find eigenvectors corresponding to each eigenvalue found in part (a). (4 marks)

3. A transformation T from the z -plane to the w -plane is defined by

$$w = \frac{z+2i}{z-i}, \quad z \neq i,$$

where $z = x + iy$, $w = u + iv$ and x, y, u and v are real.

(a) Show that the circle $|z| = 1$ is mapped onto a straight line in the w -plane under T and find an equation of the line. (5 marks)

The circle $|z - (a + ib)| = r$ in the z -plane is mapped under T onto the circle $|w| = 2$ in the w -plane, where a, b and r are real.

(b) Find the values of a, b and r . (6 marks)

4. The points A , B and C with coordinates (x_{-1}, y_{-1}) , (x_0, y_0) and (x_1, y_1) respectively lie on the curve $y = f(x)$ with $x_1 - x_0 = x_0 - x_{-1} = h$.

- (a) Use the first three terms of the Taylor series expansion in ascending powers of $(x - x_0)$ to show that

$$\left(\frac{d^2y}{dx^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2}. \quad \text{(5 marks)}$$

The variable y satisfies the differential equation

$$\frac{d^2y}{dx^2} + (x+2)\frac{dy}{dx} - 3y = 0 \quad \text{with } y = 1 \text{ at } x = 0 \text{ and } y = 1.2 \text{ at } x = 0.1$$

- (b) Use the approximations $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_{-1}}{2h}$ and $\left(\frac{d^2y}{dx^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2}$ with a step length of 0.1 to estimate the value of y at $x = 0.2$

(6 marks)

5.
$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 3 \\ 4 & q & 1 \\ 1 & 2 & -1 \end{pmatrix}, \quad q \neq 4\frac{1}{4}.$$

- (a) Find \mathbf{A}^{-1} in terms of q . **(7 marks)**

- (b) Hence, or otherwise, solve the simultaneous equations

$$x - y + 3z = 1,$$

$$4x + y + z = 2,$$

$$x + 2y - z = 5,$$

showing your working clearly.

(4 marks)

Turn over

6. Given that

$$y = \sqrt{1-x^2} \arccos x,$$

(a) show that

$$(1-x^2) \frac{dy}{dx} + xy - x^2 + 1 = 0. \quad (\text{I}) \quad (5 \text{ marks})$$

(b) By differentiating equation (I) twice, or otherwise, obtain the Maclaurin expansion of $y = \sqrt{1-x^2} \arccos x$ up to and including the term in x^3 .

(8 marks)

7. The plane Π_1 has vector equation

$$\mathbf{r} = 3\mathbf{i} + \mathbf{j} - 4\mathbf{k} + \lambda(\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} + \mathbf{k}).$$

(a) Find a vector \mathbf{n} which is normal to Π_1 . (3 marks)

(b) Hence find a vector equation of Π_1 in the form $\mathbf{r} \cdot \mathbf{n} = p$. (2 marks)

(c) Find the perpendicular distance between Π_1 and the point A with position vector $2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$, giving your answer in the form $a\sqrt{6}$, where $a \in \mathbb{Q}$. (4 marks)

The plane Π_2 has equation $\mathbf{r} \cdot (\mathbf{i} + b\mathbf{j}) = -4$. The angle between Π_1 and Π_2 is 30° .

(d) Find the possible values of the constant b . (6 marks)

END