GCE Examinations

# Further Pure Mathematics Module FP3

Advanced Subsidiary / Advanced Level

## Paper D

Time: 1 hour 30 minutes

*Instructions and Information*

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

*Advice to Candidates*

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

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**1.** Given that

$$y = \frac{1}{1-x},$$

prove by induction that $\dfrac{\mathrm{d}^n y}{\mathrm{d}x^n} = \dfrac{n!}{(1-x)^{n+1}}$ for all integers $n$, $n \geq 1$. **(7 marks)**

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**2.** The variable $y$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + y + 2, \qquad y = 0 \text{ at } x = 0.$$

*(a)* Given that $y \approx 2h$ when $x = h$, use the approximation $\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \dfrac{y_1 - y_{-1}}{2h}$ once to obtain an estimate for $y$ as a function of $h$ when $x = 2h$.

**(4 marks)**

*(b)* Use the same approximation to show that an estimate for $y$ when $x = 3h$ is given by

$$y \approx 2h(2h^3 + 8h^2 + 4h + 3).$$ **(3 marks)**

*(c)* Hence find an estimate for $y$ when $x = 0.3$ **(2 marks)**

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**3.** Given that

$$z^6 - z^3\sqrt{3} + 1 = 0,$$

*(a)* find the possible values of $z^3$, giving your answers in the form $x + \mathrm{i}y$ where $x, y \in \mathbb{R}$.

**(3 marks)**

*(b)* Hence find all possible values of $z$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi \leq \theta < \pi$.

**(7 marks)**

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**4.** *(a)* Write down the first three terms of the series of $e^{x^2}$, in ascending powers of $x$.

**(2 marks)**

*(b)* Hence, or otherwise, find the series expansion, in ascending powers of $x$ up to and including the term in $x^4$, of

$$\frac{e^{x^2}}{1+2x}.$$

**(5 marks)**

*(c)* Hence find an estimate for the area of the region bounded by the $x$-axis, the lines $x = 0$ and $x = 0.2$, and the curve

$$y = \frac{e^{x^2}}{1+2x},$$

giving your answer to 3 significant figures. **(4 marks)**

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**5.** The transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is represented by the matrix $\mathbf{A}$ where

$$\mathbf{A} = \begin{pmatrix} 2 & a & 1 \\ 1 & 2 & -1 \\ 3 & 1 & 1 \end{pmatrix}.$$

*(a)* Find $\mathbf{A}^{-1}$, showing your working clearly and stating the condition for which $\mathbf{A}$ is non-singular.

**(7 marks)**

Relative to a fixed origin $O$, the transformation $T$ maps the point $P$ onto the point $Q$. When $a = -1$, $Q$ has position vector $5\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$.

*(b)* Find the position vector of $P$, showing your working clearly. **(4 marks)**

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***Turn over***

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**6.** The planes $\Pi_1$ and $\Pi_2$ are defined by the equations $2x - y + 3z = 5$ and $x + 4y + z = -2$ respectively.

*(a)* Find, to the nearest degree, the acute angle between $\Pi_1$ and $\Pi_2$. **(4 marks)**

The point $A$ has coordinates $(2, 1, -2)$.

*(b)* Find the perpendicular distance between $A$ and $\Pi_1$. **(4 marks)**

The plane $\Pi_3$ is perpendicular to $\Pi_1$ and $\Pi_2$ and the point with coordinates $(0, 4, -1)$ lies on $\Pi_3$.

*(c)* Find the equation of $\Pi_3$ in the form $ax + by + cz = d$. **(5 marks)**

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**7.** The transformation $T$ from the complex $z$-plane to the complex $w$-plane is given by

$$w = \frac{1}{z^* - 2}, \quad z \neq 2.$$

*(a)* Show that the image in the $w$-plane of the line $\text{Re}(z) = 5$ in the $z$-plane, under $T$, is a circle. Find its centre and radius.

**(7 marks)**

The region represented by $\text{Re}(z) > 5$ in the $z$-plane is transformed under $T$ into the region represented by $R$ in the $w$-plane.

*(b)* Show the region $R$ on an Argand diagram. **(3 marks)**

*(c)* Find the image in the $w$-plane under $T$ of the half-line $\arg(z - 2) = \frac{\pi}{4}$ in the the $z$-plane.

**(4 marks)**

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**END**

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