GCE Examinations

Advanced Subsidiary / Advanced Level

Further Pure Mathematics
Module FP3

Paper H

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

Written by Rosemary Smith & Shaun Armstrong

FP3 Paper H – Marking Guide

1. assume true for $n = k$ $\therefore$ $t_k = 7 - 4k$
$\therefore$ $t_{k+1} = t_k - 4 = 7 - 4k - 4 = 7 - 4(k + 1)$ — M1 A2
$\therefore$ true for $n = k + 1$ if true for $n = k$
if $n = 1$ $7 - 4n = 7 - 4 = 3$ as required $\therefore$ true for $n = 1$ — B1
$\therefore$ by induction true for $n \in \mathbb{Z}^+$ — A1 **(5)**

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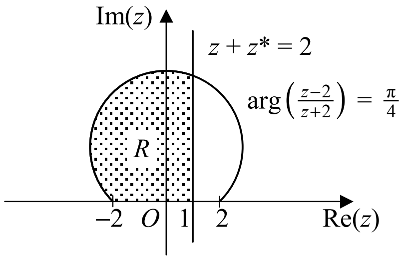
2. *(a)* (i), (ii); *(b)*

$z + z^* = 2;\ x + iy + x - iy = 2;\ 2x = 2\ \therefore x = 1$ — M1 A1



B1

B3

M1 A1 **(8)**

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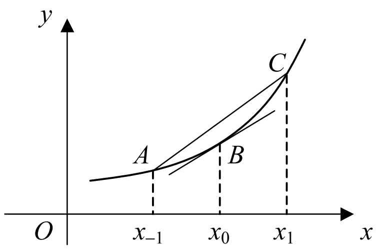
3. *(a)*



B1

grad. of tangent at $B \approx$ grad. of chord AC — M1
$\therefore\ f'(x_0) \approx \dfrac{y_1 - y_{-1}}{x_1 - x_{-1}} = \dfrac{f(x_0 + h) - f(x_0 - h)}{2h}$ — A1

(b) $f'(x_0) = \sqrt{2x_0 + f(x_0)} \approx \dfrac{f(x_1) - f(x_{-1})}{0.4}$ — M1
$f(x_1) = f(x_{-1}) + 0.4\sqrt{2x_0 + f(x_0)}$ — A1
$x_{-1} = 0,\ x_0 = 0.2,\ x_1 = 0.4;\ f(x_{-1}) = 1,\ f(x_0) = 1.25,\ f(x_1) = ?$
$\therefore\ f(x_1) = 1 + 0.4\sqrt{(0.4 + 1.25)} = 1.5138...$ — A1
$f(x_2) = f(x_0) + 0.4\sqrt{2x_1 + f(x_1)}$ — M1
$x_0 = 0.2,\ x_1 = 0.4,\ x_2 = 0.6;\ f(x_0) = 1.25,\ f(x_1) = 1.5138...,\ f(x_2) = ?$
$\therefore\ f(x_2) = 1.25 + 0.4\sqrt{(0.8 + 1.5138...)} = 1.8584...\ \therefore\ f(0.6) \approx 1.86$ (3sf) — A1 **(8)**

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4. *(a)* $\overrightarrow{AB} = -\mathbf{i} + (q+1)\mathbf{j} - 4\mathbf{k}$, $\overrightarrow{AC} = \mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ — M1 A1

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & q+1 & -4 \\ 1 & -3 & 4 \end{vmatrix}$$

$= \mathbf{i}(4q + 4 - 12) - \mathbf{j}(-4 + 4) + \mathbf{k}(3 - q - 1) = (4q - 8)\mathbf{i} + (2 - q)\mathbf{k}$ — M1 A2

(b) $\overrightarrow{AB} \times \overrightarrow{AC} = 4(q-2)\mathbf{i} + (2-q)\mathbf{k} = (q-2)[4\mathbf{i} - \mathbf{k}]$ — M1

$\therefore$ $\mathbf{n} = 4\mathbf{i} - \mathbf{k}$ is perp. to Π — A1

(c) $\mathbf{r}.(4\mathbf{i} - \mathbf{k}) = (2\mathbf{i} - \mathbf{j} + \mathbf{k}).(4\mathbf{i} - \mathbf{k}) = 8 - 1 = 7$ — M1

$\therefore$ eqn. is $\mathbf{r}.(4\mathbf{i} - \mathbf{k}) = 7$ — A1

(d) $q = -1$, $\overrightarrow{AB} \times \overrightarrow{AC} = -12\mathbf{i} + 3\mathbf{k}$

volume $= \frac{1}{6} \left| \overrightarrow{AB} \times \overrightarrow{AC} \,.\, \overrightarrow{AO} \right|$ — M1

$= \frac{1}{6} \left| (-12\mathbf{i} + 3\mathbf{k}).(-2\mathbf{i} + \mathbf{j} - \mathbf{k}) \right|$

$= \frac{1}{6} \left| 24 - 3 \right| = \frac{1}{6} \times 21 = 3.5$ units^3 — M1 A1 **(12)**

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5. *(a)* $(\cos\theta + \mathrm{i}\sin\theta)^5 = \cos 5\theta + \mathrm{i}\sin 5\theta$

$= \cos^5\theta + 5\mathrm{i}\cos^4\theta\sin\theta + 10\mathrm{i}^2\cos^3\theta\sin^2\theta + 10\mathrm{i}^3\cos^2\theta\sin^3\theta$
$+ 5\mathrm{i}^4\cos\theta\sin^4\theta + \mathrm{i}^5\sin^5\theta$ — M1 A1

equating real parts:

$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ — M1 A1

$= \cos^5\theta - 10\cos^3\theta(1 - \cos^2\theta) + 5\cos\theta(1 - \cos^2\theta)^2$ — M1

$= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos\theta - 10\cos^3\theta + 5\cos^5\theta$

$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

$= \cos\theta(16\cos^4\theta - 20\cos^2\theta + 5)$ — A1

(b) $\cos 5\theta = 0$, $5\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$ — M1

$\therefore$ $\theta = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \ldots$ — A1

if $\cos\theta(16\cos^4\theta - 20\cos^2\theta + 5) = 0$

then $\cos\theta = 0$ giving $\theta = \frac{\pi}{2}, \ldots$ or $16\cos^4\theta - 20\cos^2\theta + 5 = 0$ — M1

using quad formula, $\cos^2\theta = \frac{20 \pm \sqrt{80}}{32} = \frac{20 \pm 4\sqrt{5}}{32} = \frac{5 \pm \sqrt{5}}{8}$ — M1 A1

$\cos^2(\frac{\pi}{10}) > \cos^2(\frac{3\pi}{10})$ $\therefore$ $\cos^2(\frac{3\pi}{10}) = \frac{5 - \sqrt{5}}{8}$ — M1 A1 **(13)**

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6. *(a)* let $f(x) \equiv \ln\left(\frac{1+x}{1-2x}\right) = \ln(1+x) - \ln(1-2x)$ M1

$f'(x) = \frac{1}{1+x} - \frac{1}{1-2x}(-2) = \frac{1}{1+x} + \frac{2}{1-2x}$ M1 A1

$f''(x) = -(1+x)^{-2} - 2(1-2x)^{-2}(-2) = \frac{4}{(1-2x)^2} - \frac{1}{(1+x)^2}$ M1 A1

$f'''(x) = -8(1-2x)^{-3}(-2) + 2(1+x)^{-3} = \frac{2}{(1+x)^3} + \frac{16}{(1-2x)^3}$ A1

(b) $f(0) = 0,\ f'(0) = 1 + 2 = 3,\ f''(0) = 4 - 1 = 3,\ f'''(0) = 2 + 16 = 18$ M1 A1

$\therefore \ln\left(\frac{1+x}{1-2x}\right) = 0 + 3x + 3(\frac{1}{2!})x^2 + 18(\frac{1}{3!})x^3 + \ldots$ M1

$= 3x + \frac{3}{2}x^2 + 3x^3 + \ldots$ A1

(c) $-1 < x \leq 1$ and $-1 \leq 2x < 1 \ \therefore\ -\frac{1}{2} \leq x < \frac{1}{2}$ B1

(d) $\frac{1+x}{1-2x} = \frac{4}{3} \Rightarrow 3 + 3x = 4 - 8x$ giving $x = \frac{1}{11}$ M1 A1

$\therefore \ln\frac{4}{3} \approx \frac{3}{11} + \frac{3}{2}\times\frac{1}{121} + 3\times\frac{1}{1331} = 0.287$ (3dp) A1 **(14)**

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7. *(a)* $(\mathbf{B} - 2\mathbf{I})\mathbf{A} = \begin{pmatrix} 4 & 5 & 2 \\ -1 & -2 & -2 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 2 & a & 2 \\ -1 & b & -2 \\ 0 & 0 & c \end{pmatrix} = 3\mathbf{I} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ M1

$\therefore\ 4a + 5b + 0 = 0$

$-a - 2b + 0 = 3$ M1 A1

solve. simul. giving $a = 5,\ b = -4$ M1 A1

$0 + 0 + 3c = 3,\ c = 1$ A1

(b) $\frac{1}{3}(\mathbf{B} - 2\mathbf{I})\mathbf{A} = \mathbf{I}\ \therefore\ \mathbf{A}^{-1} = \frac{1}{3}(\mathbf{B} - 2\mathbf{I})$ M1

$= \frac{1}{3}\begin{pmatrix} 4 & 5 & 2 \\ -1 & -2 & -2 \\ 0 & 0 & 3 \end{pmatrix}$ A1

(c) $\begin{pmatrix} 2 & 5 & 2 \\ -1 & -4 & -2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ M1

$\therefore\ 2x + 5y + 2z = x$

$-x - 4y - 2z = y$

$z = z$ A1

giving $x + 5y + 2z = 0$ M1 A1

(d) $\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 4 & 5 & 2 \\ -1 & -2 & -2 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} -3 \\ 0 \\ 9 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}$ M1 A1

$p = -1,\ q = 0,\ r = 3$ A1 **(15)**

Total **(75)**

Performance Record – FP3 Paper H

Question no.	1	2	3	4	5	6	7	Total
Topic(s)	proof by induction	complex loci	step-by-step soln. of diff. eqn.	vectors	complex nos., De Moivre's	Maclaurin series	matrices, inverse	
Marks	5	8	8	12	13	14	15	75
Student								