

GCE Examinations
Advanced Subsidiary / Advanced Level
Further Pure Mathematics
Module FP3

Paper F

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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FP3 Paper F – Marking Guide

1. assume true for $n = k$ $\therefore \sum_{r=1}^k \ln \frac{r+1}{r} = \ln(k+1)$
- $\therefore \sum_{r=1}^{k+1} \ln \frac{r+1}{r} = \ln(k+1) + \ln \frac{k+2}{k+1}$ M1 A1
- $= \ln \frac{(k+1)(k+2)}{k+1} = \ln(k+2)$ M1
- $= \ln[(k+1)+1]$ A1
- \therefore true for $n = k+1$ if true for $n = k$
- if $n = 1$ $\sum_{r=1}^n \ln \frac{r+1}{r} = \ln \frac{2}{1} = \ln 2$, $\ln(n+1) = \ln 2$ \therefore true for $n = 1$ B1
- \therefore by induction true for $n \in \mathbb{Z}^+$ A1 **(6)**
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2. (a) $\begin{vmatrix} 2-\lambda & 3 \\ 3 & -6-\lambda \end{vmatrix} = 0$ M1
- $\therefore (2-\lambda)(-6-\lambda) - 9 = 0$ A1
- $\lambda^2 + 4\lambda - 21 = 0$
- $(\lambda+7)(\lambda-3) = 0 \therefore \lambda = -7 \text{ or } 3$ M1 A1
- (b) $\lambda = 3$, $\begin{pmatrix} -1 & 3 \\ 3 & -9 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $-x+3y=0 \therefore 3y=x$ \therefore eigenvector $k \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ M1 A1
- $\lambda = -7$, $\begin{pmatrix} 9 & 3 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $3x+y=0 \therefore y=-3x$ \therefore eigenvector $k \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ M1 A1 **(8)**
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3. (a) $w(z-i) = z+2i$; $wz - wi = z+2i$ M1
- $z(w-1) = wi+2i \therefore z = \frac{i(w+2)}{w-1}$ A1
- $|z|=1 \therefore |i||w+2| = |w-1|$ M1
- $|w+2| = |w-1|$ A1
- \therefore perp. bisector of $-2+0i$ and $1+0i \therefore u = -\frac{1}{2}$ A1
- (b) $|w|=2$, $\left| \frac{z+2i}{z-i} \right| = 2$
- $\therefore |z+2i| = 2|z-i|$ M1
- $x^2 + (y+2)^2 = 4x^2 + 4(y-1)^2$ A1
- $x^2 + y^2 + 4y + 4 = 4x^2 + 4y^2 - 8y + 4$
- $3x^2 + 3y^2 - 12y = 0$
- $x^2 + y^2 - 4y = 0$ A1
- $x^2 + (y-2)^2 - 4 = 0$ or $x^2 + (y-2)^2 = 4$ M1
- \therefore circle, centre $0+2i$, radius 2 A1
- $a=0, b=2, r=2$ A1 **(11)**
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4. (a) $y = y_0 + (x - x_0) \left(\frac{dy}{dx} \right)_0 + \frac{1}{2} (x - x_0)^2 \left(\frac{d^2y}{dx^2} \right)_0 + \dots$ B1
- $x = x_0 + h, y_1 \approx y_0 + h \left(\frac{dy}{dx} \right)_0 + \frac{1}{2} h^2 \left(\frac{d^2y}{dx^2} \right)_0$ (I) M1 A1
- $x = x_0 - h, y_{-1} \approx y_0 - h \left(\frac{dy}{dx} \right)_0 + \frac{1}{2} h^2 \left(\frac{d^2y}{dx^2} \right)_0$ (II) M1 A1
- (I) + (II) $y_1 + y_{-1} \approx 2y_0 + h^2 \left(\frac{d^2y}{dx^2} \right)_0$ giving $\left(\frac{d^2y}{dx^2} \right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2}$ M1 A1
- (b) $\frac{d^2y}{dx^2} + (x + 2) \frac{dy}{dx} - 3y = 0$
- $\frac{y_1 - 2y_0 + y_{-1}}{0.01} + (x_0 + 2) \frac{y_1 - y_{-1}}{0.2} - 3y_0 = 0$ M1 A1
- $x_{-1} = 0, x_0 = 0.1, x_1 = 0.2; y_{-1} = 1, y_0 = 1.2, y_1 = ?$
- $100(y_1 - 2.4 + 1) + 5(0.1 + 2)(y_1 - 1) - 3.6 = 0$ M1 A1
- giving $110.5y_1 = 154.1 \therefore y_1 = 1.39457\dots = 1.39$ (3sf) M1 A1 (11)
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5. (a) $\det \mathbf{A} = 1(-q - 2) + 1(-4 - 1) + 3(8 - q) = 17 - 4q$ M1 A1
- matrix of cofactors: $\begin{pmatrix} -q-2 & 5 & 8-q \\ 5 & -4 & -3 \\ -1-3q & 11 & q+4 \end{pmatrix}$ M1 A2
- $\therefore \mathbf{A}^{-1} = \frac{1}{17-4q} \begin{pmatrix} -q-2 & 5 & -1-3q \\ 5 & -4 & 11 \\ 8-q & -3 & q+4 \end{pmatrix}$ M1 A1
- (b) $\begin{pmatrix} 1 & -1 & 3 \\ 4 & 1 & 1 \\ 1 & 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} \quad q = 1$ M1
- $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{13} \begin{pmatrix} -3 & 5 & -4 \\ 5 & -4 & 11 \\ 7 & -3 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} = \frac{1}{13} \begin{pmatrix} -13 \\ 52 \\ 26 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix}$ M1 A1
- $x = -1, y = 4, z = 2$ A1 (11)
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6. (a) $\frac{dy}{dx} = \sqrt{1-x^2} \times \frac{-1}{\sqrt{1-x^2}} + \arccos x \times \frac{1}{2} \frac{1}{\sqrt{1-x^2}} (-2x)$ M1 A1
- $\frac{dy}{dx} = -1 - \frac{y}{\sqrt{1-x^2}} \times \frac{x}{\sqrt{1-x^2}}$ M1
- $(1-x^2) \frac{dy}{dx} = -(1-x^2) - xy$ M1
- $(1-x^2) \frac{dy}{dx} + xy - x^2 + 1 = 0$ A1
- (b) $(1-x^2) \frac{d^2y}{dx^2} + \frac{dy}{dx} (-2x) + x \frac{dy}{dx} + y - 2x = 0$ M1 A1
- $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y - 2x = 0$
- $(1-x^2) \frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} (-2x) - x \frac{d^2y}{dx^2} - \frac{dy}{dx} + \frac{dy}{dx} - 2 = 0$ M1 A1
- $(1-x^2) \frac{d^3y}{dx^3} - 3x \frac{d^2y}{dx^2} - 2 = 0$
- $y_0 = 1 \times \arccos 0 = \frac{\pi}{2}; 1 \left(\frac{dy}{dx} \right)_0 + 0 - 0 + 1 = 0 \therefore \left(\frac{dy}{dx} \right)_0 = -1$ A1
- $1 \left(\frac{d^2y}{dx^2} \right)_0 - 0 + \frac{\pi}{2} - 0 = 0 \therefore \left(\frac{d^2y}{dx^2} \right)_0 = -\frac{\pi}{2}$
- $1 \left(\frac{d^3y}{dx^3} \right)_0 - 0 - 2 = 0 \therefore \left(\frac{d^3y}{dx^3} \right)_0 = 2$ A1
- $\therefore y = \frac{\pi}{2} - x - \frac{\pi}{4}x^2 + \frac{1}{3}x^3 + \dots$ M1 A1 (13)
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7. (a) $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{vmatrix}$
- $\mathbf{n} = \mathbf{i}(1-2) - \mathbf{j}(0-2) + \mathbf{k}(0-1) = -\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ M1 A2
- (b) $\mathbf{r} \cdot (-\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = (3\mathbf{i} + \mathbf{j} - 4\mathbf{k}) \cdot (-\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = -3 + 2 + 4 = 3$ M1 A1
- $\mathbf{r} \cdot (-\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = 3$
- (c) $l_1 : \mathbf{r} \cdot \frac{-\mathbf{i} + 2\mathbf{j} - \mathbf{k}}{\sqrt{6}} = \frac{3}{\sqrt{6}}$ B1
- plane parallel to l_1 through A :
- $\mathbf{r} \cdot (-\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = (2\mathbf{i} + \mathbf{j} + 4\mathbf{k}) \cdot (-\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = -2 + 2 - 4 = -4$ M1
- $\therefore \mathbf{r} \cdot \frac{-\mathbf{i} + 2\mathbf{j} - \mathbf{k}}{\sqrt{6}} = \frac{-4}{\sqrt{6}}$ A1
- \therefore distance A to $l_1 = \frac{7}{\sqrt{6}} = \frac{7}{6}\sqrt{6}$ A1
- (d) $|(-\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \cdot (\mathbf{i} + b\mathbf{j})| = \sqrt{6}\sqrt{(1+b^2)}\cos 30^\circ$ M1 A1
- $|-1 + 2b| = \sqrt{6}\sqrt{(1+b^2)}\frac{\sqrt{3}}{2}$ A1
- $(2b-1)^2 = \frac{18}{4}(1+b^2)$ M1
- $2(4b^2 - 4b + 1) = 9(1+b^2)$
- giving $b^2 + 8b + 7 = 0$
- $(b+1)(b+7) = 0 \therefore b = -1$ or -7 M1 A1 (15)
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Total (75)

