

GCE Examinations  
Advanced Subsidiary / Advanced Level  
**Further Pure Mathematics**  
**Module FP3**

Paper E

**MARKING GUIDE**

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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## FP3 Paper E – Marking Guide

1. (a)  $|z - (-1 + i)| = 2$  or  $|z + 1 - i| = 2$  M1 A1
- (b)  $y = 0 \therefore |x + 1 - i| = 2$  M1  
 $(x + 1)^2 + 1 = 4$  A1  
giving  $x = -1 \pm \sqrt{3}$  A1 (5)
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2. assume true for  $n = k \therefore 2^k > 2k$   
 $\therefore 2^{k+1} > 2 \times 2k$  M1 A1  
 $\therefore 2^{k+1} > 2k + 2k$   
 $\therefore 2^{k+1} > 2k + 2$  as  $2k > 2$  for  $k \geq 3$  M1  
 $\therefore 2^{k+1} > 2(k + 1)$  A1  
 $\therefore$  true for  $n = k + 1$  if true for  $n = k$   
if  $n = 3, 2^3 = 8, 2 \times 3 = 6 \therefore 2^3 > 2 \times 3$  so true for  $n = 3$  B1  
 $\therefore$  by induction true for integers  $n, n \geq 3$  A1 (6)
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3. (a)  $\ln(1 + 2x) - 2xe^{-x} = 2x - \frac{1}{2}(2x)^2 + \frac{1}{3}(2x)^3 + \dots - 2x(1 - x + \frac{1}{2!}x^2 + \dots)$  M1 A1  
 $= 2x - 2x^2 + \frac{8}{3}x^3 - 2x + 2x^2 - x^3 + \dots$  M1  
 $\therefore \ln(1 + 2x) \approx \frac{5}{3}x^3$  so  $A = \frac{5}{3}$  A1
- (b)  $\lim_{x \rightarrow 0} \left( \frac{\ln(1+2x) - 2xe^{-x}}{x^3} \right) = \lim_{x \rightarrow 0} \left( \frac{\frac{5}{3}x^3 + kx^4 + \dots}{x^3} \right) = \frac{5}{3}$  M1 A1 (6)
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4. (a)  $\begin{pmatrix} 2 & -1 & 1 \\ 0 & 1 & -1 \\ -3 & 3 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 1 \times \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \therefore \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$  is eigenvector, eigenvalue = 1 M1 A1
- (b)  $\begin{vmatrix} 2-\lambda & -1 & 1 \\ 0 & 1-\lambda & -1 \\ -3 & 3 & 1-\lambda \end{vmatrix} = 0$  M1  
 $(2 - \lambda)[(1 - \lambda)^2 + 3] + 1(0 - 3) + 1[0 + 3(1 - \lambda)] = 0$  A1  
 $(2 - \lambda)(1 - \lambda)^2 + 6 - 3\lambda - 3 + 3 - 3\lambda = 0$  M1  
 $(2 - \lambda)(1 - \lambda)^2 + 6(1 - \lambda) = 0$   
 $(1 - \lambda)[(2 - \lambda)(1 - \lambda) + 6] = 0$  A1  
 $\therefore \lambda = 1$  or  $\lambda^2 - 3\lambda + 8 = 0$  M1  
“ $b^2 - 4ac$ ” =  $9 - 32 = -23 (< 0) \Rightarrow$  no real roots  
 $\therefore$  only 1 real eigenvalue A1 (8)
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5. (a)  $\text{Im}(z) = 2 \therefore y = 2$   
 $u + iv = (x + 2i)^2 = x^2 + 4xi - 4$  M1 A1  
 $u = x^2 - 4, v = 4x$  M1  
 $x = \frac{v}{4}$   
 $\therefore u = \frac{v^2}{16} - 4$  or  $v^2 = 16(u + 4)$  which is a parabola M1 A1
- (b)  $\arg w = \arg(z^2) = 2 \arg z \therefore \arg w = \frac{\pi}{2}$  M1 A1
- (c)  $\arg w = \frac{\pi}{2} \Rightarrow u = 0, v \geq 0$  M1  
 $\therefore v^2 = 16(0 + 4) = 64, v \geq 0 \therefore v = 8$  M1 A1  
 $P$  represents  $0 + 8i$  A1 **(11)**
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6. (a) (i)  $\frac{d^2 y}{dx^2} = 2x + \frac{dy}{dx} \cos x - y \sin x$  M1 A1  
 $\frac{d^3 y}{dx^3} = 2 + \frac{d^2 y}{dx^2} \cos x - \frac{dy}{dx} \sin x - \frac{dy}{dx} \sin x - y \cos x$  M1 A2
- (ii)  $y_0 = 1, \left(\frac{dy}{dx}\right)_0 = 1, \left(\frac{d^2 y}{dx^2}\right)_0 = 1, \left(\frac{d^3 y}{dx^3}\right)_0 = 2$  A1  
 $\therefore y = 1 + x + \frac{1}{2!}x^2 + 2\left(\frac{1}{3!}\right)x^3 + \dots = 1 + x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \dots$  M1 A1
- (iii)  $x = -0.1, y \approx 0.904666\dots = 0.905$  (3sf) M1 A1
- (b)  $\frac{y_1 - y_{-1}}{0.2} = x_0^2 + y_0 \cos x_0$  M1  
 $y_1 = 0.2(x_0^2 + y_0 \cos x_0) + y_{-1}$  A1  
 $x_{-1} = -0.1, x_0 = 0, x_1 = 0.1; y_{-1} = 0.904666\dots, y_0 = 1, y_1 = ?$   
 $y_1 = 0.2(0 + 1) + 0.904666\dots = 1.104666\dots = 1.10$  (3sf) A1 **(13)**
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7. (a)  $\vec{AB} = 2\mathbf{i} + \mathbf{j} + 5\mathbf{k}$ ,  $\vec{AC} = -\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$  M1 A1
- $$\vec{AB} \times \vec{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 5 \\ -1 & -2 & -4 \end{vmatrix}$$
- $= \mathbf{i}(-4 + 10) - \mathbf{j}(-8 + 5) + \mathbf{k}(-4 + 1) = 6\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$  M1 A1
- $\mathbf{n} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$
- eqn. of plane is  $\mathbf{r} \cdot (2\mathbf{i} + \mathbf{j} - \mathbf{k}) = (\mathbf{i} + \mathbf{j}) \cdot (2\mathbf{i} + \mathbf{j} - \mathbf{k}) = 2 + 1 + 0 = 3$  M1 A1
- $\mathbf{r} \cdot (2\mathbf{i} + \mathbf{j} - \mathbf{k}) = 3$
- (b)  $\mathbf{r} = -2\mathbf{i} - 5\mathbf{j} + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$  A1
- (c) at E  $[(-2 + 2\lambda)\mathbf{i} + (-5 + \lambda)\mathbf{j} - \lambda\mathbf{k}] \cdot (2\mathbf{i} + \mathbf{j} - \mathbf{k}) = 3$  M1
- $-4 + 4\lambda - 5 + \lambda + \lambda = 3$  giving  $\lambda = 2$  M1 A1
- $\therefore E$  is  $(2, -3, -2)$  A1
- (d)  $E$  is midpoint of  $DF$ ;  $D(-2, -5, 0)$   $E(2, -3, -2)$   $\therefore F(6, -1, -4)$  M1 A1 (13)

8. (a)  $\det \mathbf{M} = 2(0 - 2) - 1(0 - 2) - 1(0 - 6) = -4 + 2 + 6 = 4$  M1 A1
- matrix of cofactors:  $\begin{pmatrix} -2 & 2 & -6 \\ -2 & 2 & -2 \\ 4 & -2 & 6 \end{pmatrix}$  M1 A1
- $\therefore \mathbf{M}^{-1} = \frac{1}{4} \begin{pmatrix} -2 & -2 & 4 \\ 2 & 2 & -2 \\ -6 & -2 & 6 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -1 & -1 & 2 \\ 1 & 1 & -1 \\ -3 & -1 & 3 \end{pmatrix}$  M1 A1
- (b)  $\frac{x-1}{3} = \frac{y+1}{-3} = \frac{z}{4} = t$
- $\therefore x = 3t + 1, y = -3t - 1, z = 4t$  M1 A1
- $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -1 & -1 & 2 \\ 1 & 1 & -1 \\ -3 & -1 & 3 \end{pmatrix} \begin{pmatrix} 3t+1 \\ -3t-1 \\ 4t \end{pmatrix}$$
- $$= \frac{1}{2} \begin{pmatrix} -3t-1+3t+1+8t \\ 3t+1-3t-1-4t \\ -9t-3+3t+1+12t \end{pmatrix} = \begin{pmatrix} 4t \\ -2t \\ 3t-1 \end{pmatrix}$$
- M1 A2
- $\therefore x = 4t, y = -2t, z = 3t - 1$
- giving  $(t =) \frac{x}{4} = \frac{y}{-2} = \frac{z+1}{3}$  M1 A1 (13)

Total (75)

