Paper Reference(s)

6676

Edexcel GCE

Pure Mathematics P6

Advanced/Advanced Subsidiary

Monday 23 June 2003 − Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P6), the paper reference (6676), your surname, other name and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
This paper has eight questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. Referred to a fixed origin O, the position vectors of three non-collinear points A, B and C are $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively. By considering $\overrightarrow{AB} \times \overrightarrow{AC}$, prove that the area of ΔABC can be expressed in the form $\frac{1}{2}\left| \mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a} \right|$.

(5)

2.

$$\mathrm{f}(n) = (2n + 1)7^n - 1.$$

Prove by induction that, for all positive integers n, $\mathrm{f}(n)$ is divisible by 4.

(6)

3.

$$\mathbf{M} = \begin{pmatrix} 4 & -5 \\ 6 & -9 \end{pmatrix}$$

(*a*) Find the eigenvalues of $\mathbf{M}$.

(4)

A transformation T: $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ is represented by the matrix $\mathbf{M}$. There is a line through the origin for which every point on the line is mapped onto itself under T.

(*b*) Find a cartesian equation of this line.

(3)

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4. (i) (*a*) On the same Argand diagram sketch the loci given by the following equations.

$$|z-1| = 1,$$

$$\arg(z+1) = \frac{\pi}{12},$$

$$\arg(z+1) = \frac{\pi}{2}.$$

(4)

(*b*) Shade on your diagram the region for which

$$|z-1| \leq 1 \quad \text{and} \quad \frac{\pi}{12} \leq \arg(z+1) \leq \frac{\pi}{2}.$$

(1)

(ii) (*a*) Show that the transformation

$$w = \frac{z-1}{z}, \quad z \neq 0,$$

maps $|z-1| = 1$ in the z-plane onto $|w| = |w-1|$ in the w-plane.

(3)

The region $|z-1| \leq 1$ in the z-plane is mapped onto the region T in the w-plane.

(*b*) Shade the region T on an Argand diagram.

(2)

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5. (*a*) Use de Moivre's theorem to show that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$$

(6)

(*b*) Hence find 3 distinct solutions of the equation $16x^5 - 20x^3 + 5x + 1 = 0$, giving your answers to 3 decimal places where appropriate.

(4)

6.
$$\mathbf{A} = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 1 & 1 \\ 5 & 3 & u \end{pmatrix}, \quad u \neq 1.$$

(*a*) Show that det $\mathbf{A} = 2(u - 1)$.

(2)

(*b*) Find the inverse of $\mathbf{A}$.

(6)

The image of the vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ when transformed by the matrix $\begin{pmatrix} 3 & 1 & -1 \\ 1 & 1 & 1 \\ 5 & 3 & 6 \end{pmatrix}$ is $\begin{pmatrix} 3 \\ 1 \\ 6 \end{pmatrix}$.

(*c*) Find the values of a, b and c.

(3)

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7. The plane Π_1 passes through the P, with position vector $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, and is perpendicular to the line L with equation

$$\mathbf{r} = 3\mathbf{i} - 2\mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}).$$

(*a*) Show that the Cartesian equation of Π_1 is $x - 5y - 3z = -6$.

(4)

The plane Π_2 contains the line L and passes through the point Q, with position vector $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$.

(*b*) Find the perpendicular distance of Q from Π_1.

(4)

(*c*) Find the equation of Π_2 in the form $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$.

(4)

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8. $$\frac{dy}{dx} = x^2 - y^2, \quad y = 1 \text{ at } x = 0. \quad \text{(I)}$$

(*a*) Use the approximation $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_0}{h}$ with a step length of 0.05 to estimate the values of y at $x = 0.05$ and $x = 0.1$.

(6)

(*b*) By differentiating (I) twice with respect to x, show that

$$\frac{d^3y}{dx^3} + 2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 - 2 = 0.$$

(4)

(*c*) Hence, for (I), find the series solution for y in ascending powers of x up to and including the term in x^3.

(4)

END