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Mark Scheme (Results)

Summer 2009

GCE

GCE Mathematics (6676/01)

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June 2009
6676 Further Pure Mathematics FP3 (legacy)
Mark Scheme

Question Number	Scheme	Marks
Q1	At $x = 0.1$, $y_1 = 0.1(0 \text{ x } 0 + 3) + 0 = 0.3$	B1
	$x = 0.2$, $y_2 = 0.1\ (0.1 \text{ x } 0.3^2 + 3) + 0.3$	M1
	$(= 0.3009 + 0.3)$	
	$= 0.6009$	A1
	$x = 0.3$, $y_3 = 0.1\ (0.2 \text{ x } 0.6009^2 + 3) + 0.6009$	M1
	$(= 0.307221616........+ 0.6009)$	
	$= 0.908(121616.......)$ Allow awrt 0.908	A1
		[5]
Q2 (a)	$\mathbf{b} \times \mathbf{c} = 0\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$	M1 A1 A1
		(3)
(b)	$\mathbf{a}.(\mathbf{b} \times \mathbf{c}) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} . \begin{pmatrix} 0 \\ 5 \\ 5 \end{pmatrix} = 0 + 5 + 0 = 5$	M1 A1ft
		(2)
(c)	Area of triangle OBC = $\frac{1}{2}\lvert 5\mathbf{j} + 5\mathbf{k} \rvert = \frac{5}{2}\sqrt{2}$ oe	M1 A1
		(2)
(d)	Volume of tetrahedron = $\frac{1}{6} \times 5 = \frac{5}{6}$	B1 ft
		(1)
		[8]

Question Number	Scheme	Marks
Q3 (a)	$\begin{vmatrix} 6-\lambda & 1 & -1 \\ 0 & 7-\lambda & 0 \\ 3 & -1 & 2-\lambda \end{vmatrix} = 0$	
	$\therefore (6-\lambda)((7-\lambda)(2-\lambda)-0)-1\times 0-1(0-3(7-\lambda))=0$ $\therefore (6-\lambda)(7-\lambda)(2-\lambda)+3(7-\lambda)=0$	M1
	$(7-\lambda)=0$ verifies $\lambda = 7$ is an eigenvalue	M1
	They may show $\lambda = 7$ in the determinant (e.g. -1(0 – 0)-1(0 – 0)-1(0 – 0)) $\therefore (7-\lambda)\{12-8\lambda+\lambda^2+3\}=0$ $\therefore (7-\lambda)\{\lambda^2-8\lambda+15\}=0$ (NB $\therefore \lambda^3-15\lambda^2+71\lambda-105=0$)	A1
	$\therefore (7-\lambda)(\lambda-5)(\lambda-3)=0$ and 3 and 5 are the other two eigenvalues	M1 A1 (5)
(b)	$\begin{pmatrix} 6 & 1 & -1 \\ 0 & 7 & 0 \\ 3 & -1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 7\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or $\begin{pmatrix} -1 & 1 & -1 \\ 0 & 0 & 0 \\ 3 & -1 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	M1
	$-x+y-z=0$ $(0=0)$ $3x-y-5z=0$ Solves to obtain $x = 3z$ and $y = 4z$ (3y = 4x) or equivalent	M1 A1
	Obtains eigenvector as $\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$ (or multiple)	A1
		(4) **[9]**

Question Number	Scheme	Marks
Q4 (a)	$\frac{d^2y}{dx^2}+2\times2+1=1$, and so $\frac{d^2y}{dx^2}=-4$ at $x=0$.	B1
	$y'''+\{(1+y^2)y''+2y(y')(y')\}+y'=2e^{2x}$	M1 {M1 A1} A1
	$y'''+(1+1)(-4)+2\times1(2)(2)+2=2$, i.e. $y'''=0$	B1 cso
		(6)
(b)	$y=1,+2x(+...)$	B1,B1
	$-\frac{4x^2}{2}+\frac{0x^3}{6}+\frac{40x^4}{24}$	M1 A1
	$(=-2x^2+\frac{5x^4}{3})$	
		(4)
		[10]
Q5 (a)	$\cos6\theta=\mathrm{Re}[(\cos\theta+i\sin\theta)^6]$	M1
	$(\cos\theta+\mathrm{i}\sin\theta)^6=c^6+6c^5\mathrm{i}s+15c^4\mathrm{i}^2s^2+20c^3\mathrm{i}^3s^3+15c^2\mathrm{i}^4s^4+6c\mathrm{i}^5s^5+\mathrm{i}^6s^6$	
	$\cos6\theta=c^6-15c^4s^2+15c^2s^4-s^6$	M1 A1
	$=c^6-15c^4(1-c^2)+15c^2(1-c^2)^2-(1-c^2)^3$	M1
	$\cos6\theta=c^6-15c^4+15c^6+15c^2(1-2c^2+c^4)\ -(1-3c^2+3c^4-c^6)$	
	$\cos6\theta=32\cos^6\theta-48\cos^4\theta+18\cos^2\theta-1$ *	A1
		(5)
(b)	$\cos6\theta=\cos2\theta\ \rightarrow 32\cos^6\theta-48\cos^4\theta+18\cos^2\theta-1=2\cos^2\theta-1$	M1 A1
	$32\cos^6\theta-48\cos^4\theta+16\cos^2\theta=0$	
	$16\cos^2\theta(2\cos^4\theta-3\cos^2\theta+1)=0$	M1
	$(2\cos^2\theta-1)(\cos^2\theta-1)=0$	
	$\therefore\cos^2\theta=0,\ \frac{1}{2}$ or 1 so $\cos\theta=0,\ \pm\frac{1}{\sqrt{2}}$ or ±1	M1
	Uses arccos on at least 3 different values	
	$\therefore\theta=0,\frac{\pi}{4},\frac{\pi}{2},\frac{3\pi}{4}$ and π	A1,A1
	Decimals: Allow 0, 0.785, 1.57, 2.36, 3.14 (awrt)	
	3correct solutions A1, all correct A1	
		(6)
		[11]

Question Number	Scheme	Marks
Q6 (a)	When $n = 1$ LHS = RHS = $\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$. Result true for $n = 1$	B1
	Assume result true for $n = k$ i.e. $\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}^k = \begin{pmatrix}\cos k\theta & -\sin k\theta\\ \sin k\theta & \cos k\theta\end{pmatrix}$	
	And multiply both sides by $\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$	
	Then $\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}^{k+1} = \begin{pmatrix}\cos k\theta & -\sin k\theta\\ \sin k\theta & \cos k\theta\end{pmatrix}\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$	M1
	$= \begin{pmatrix}\cos k\theta\cos\theta - \sin k\theta\sin\theta & -\cos k\theta\sin\theta - \sin k\theta\cos\theta\\ \sin k\theta\cos\theta + \cos k\theta\sin\theta & -\sin k\theta\sin\theta + \cos k\theta\cos\theta\end{pmatrix}$	M1
	i.e. $\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}^{k+1} = \begin{pmatrix}\cos(k+1)\theta & -\sin(k+1)\theta\\ \sin(k+1)\theta & \cos(k+1)\theta\end{pmatrix}$	A1
	Conclude, that by induction result is true for all positive integers	B1 cso (5)
(b)	When $n = 1$, $\mathrm{f}(n) = 7\times5-3 = 32$, which is divisible by 16, so result true for $n = 1$	B1
	Consider $\mathrm{f}(k+1)-\mathrm{f}(k) = (4k+7)5^{k+1}-(4k+3)5^k$	M1
	$= 5^k(20k+35-4k-3)$	M1
	$= 5^k(16k+32)$, which is divisible by 16	A1
	If f(k) is divisible by 16, then this implies f(k+1) is also divisible by 16 Thus by induction f(n) is divisible by 16 for all positive integers n.	B1 cso (5)
		[10]

Question Number	Scheme	Marks
Q7 (a)	If the lines meet, $-1+3\lambda=-4+3\mu$ and $2+4\lambda=2\mu$	M1
	Solve to give $\lambda=0(\mu=1)$	M1 A1
	Also $1-\lambda=\alpha$ and so $\alpha=1$.	B1 (4)
(b)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 3 & 4 \\ 0 & 3 & 2 \end{vmatrix} = \begin{pmatrix} -6 \\ 2 \\ -3 \end{pmatrix}$	M1 A1
	$\boldsymbol{r}\bullet\begin{pmatrix} -6 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \\ -3 \end{pmatrix}\bullet(e.g.\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}) = -14$	M1
	Hence $-6x + 2y - 3z + 14 = 0$	A1 (4)
(c)	$\pm(\mathbf{a}_1-\mathbf{a}_2)=\pm(\mathbf{i}-3\mathbf{j}-2\mathbf{k})$	M1
	$\left\lvert\frac{(\mathbf{a}_1-\mathbf{a}_2)\bullet\mathbf{n}}{\lvert\mathbf{n}\rvert}\right\rvert = \left\lvert\frac{(\mathbf{i}-3\mathbf{j}-2\mathbf{k})\bullet(-6\mathbf{i}+2\mathbf{j}-3\mathbf{k})}{\lvert-6\mathbf{i}+2\mathbf{j}-3\mathbf{k}\rvert}\right\rvert = \left\lvert\frac{-6-6+6}{\sqrt{6^2+2^2+3^2}}\right\rvert$	M1
	Distance is $\frac{6}{7}$	A1 cso (3)
		[11]

Question Number	Scheme	Marks
Q8 (a)	$\sqrt{\{(x-3)^2+y^2\}} = 2\sqrt{\{x^2+(y-4)^2\}}$ or $(x-3)^2+y^2 = 4\{x^2+(y-4)^2\}$	M1
	$3x^2+3y^2+6x-32y+55=0$	A1
	$(x+1)^2+(y-\frac{16}{3})^2=\frac{100}{9}$	M1
	Centre is (–1, 16/3) and radius is 10/3	A1,A1,A1 cso
		(6)
(b)	$w=\frac{12}{z} \to z=\frac{12}{w}$, and so $\left\|\frac{12}{w}-3\right\| = 2\left\|\frac{12}{w}-4\text{i}\right\|$ substituting for z	M1
	$\|3w-12\| = 2\|4\text{i}w-12\|$ multiplication by $\|w\|$ or equivalent	M1
	$\|w-4\| = \frac{8}{3}\|w+3\text{i}\|$ obtains the locus of Q in the required form	M1, A2, 1, 0
	A2 if completely correct deduct 1 for each error on their a, k or b	
		(5)
		[11]