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Mark Scheme (Results)
Summer 2007

GCE

GCE Mathematics

Further Pure Mathematics FP3 (6676)

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June 2007
6676 Further Pure Mathematics FP3
Mark Scheme

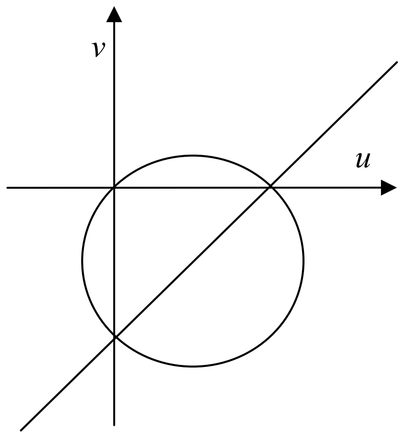
Question Number	Scheme	Marks
1.	(a) $\frac{y_1 - 0.2}{0.1} \approx \left(\frac{dy}{dx}\right)_0 = 0.2 \times e^0 \quad (= 0.2)$	M1
	$y_1 \approx 0.22$	A1 **(2)**
	(b) $\left(\frac{dy}{dx}\right)_1 \approx 0.22 \times e^{0.01} \approx 0.2222\ \ldots$	B1
	$\frac{y_2 - 0.2}{0.2} \approx 0.2222\ldots$	M1
	$y_2 \approx 0.2444$ cao	A1 **(3)**
		[5]
2.	(a) $(1-x^2)\frac{d^3y}{dx^3} - 2x\frac{d^2y}{dx^2} - x\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2\frac{dy}{dx} = 0$	M1
	At $x = 0$, $\frac{d^3y}{dx^3} = -\frac{dy}{dx} = 1$	M1 A1 cso **(3)**
	(b) $\left(\frac{d^2y}{dx^2}\right)_0 = -4$ Allow anywhere	B1
	$y = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{6}x^3 + \ldots$	
	$= 2 - x - 2x^2, + \frac{1}{6}x^3 + \ldots$	M1 A1ft, A1 (dep)**(4)**
		[7]

Question Number	Scheme	Marks
3.	(a) $\begin{pmatrix} 3 & 4 & p \\ -1 & q & -4 \\ 1 & 1 & 3 \end{pmatrix} . \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \lambda \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$	
	Third row $1-3=-\lambda \Rightarrow \lambda=2$	M1 A1 **(2)**
	(b) $\begin{pmatrix} 3 & 4 & p \\ -1 & q & -4 \\ 1 & 1 & 3 \end{pmatrix} . \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 4-p \\ q+4 \\ -2 \end{pmatrix} = 2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$	M1 A1
	First row $4-p=0 \Rightarrow p=4$ Method for either	M1
	Second row $q+4=2 \Rightarrow q=-2$ Both correct	A1 ft **(4)**
	(c) $\begin{pmatrix} 3 & 4 & 4 \\ -1 & -2 & -4 \\ 1 & 1 & 3 \end{pmatrix} . \begin{pmatrix} l \\ m \\ n \end{pmatrix} = \begin{pmatrix} 10 \\ -4 \\ 3 \end{pmatrix}$	
	$3l+4m+4n=10$ $-l-2m-4n=-4$ $l+m+3n=3$ Obtaining 3 linear equations	M1
	$2l+2m=6$ $3l+2m=8$ Reducing to a pair of equations and solving for one variable	M1
	$l=2, m=1, n=0$ Solving for all three variables.	M1 A1 **(4)** **[10]**
	Alternative to (c)	
	$\mathbf{A}^{-1} = \frac{1}{6}\begin{pmatrix} 2 & 8 & 8 \\ -1 & -5 & -8 \\ -1 & -1 & 2 \end{pmatrix}$	M1 M1
	$\frac{1}{6}\begin{pmatrix} 2 & 8 & 8 \\ -1 & -5 & -8 \\ -1 & -1 & 2 \end{pmatrix} . \begin{pmatrix} 10 \\ -4 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$	M1 A1 **(4)**

Question Number	Scheme	Marks
4.	(a) $z^n = (\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$	
	$z^{-n} = (\cos\theta + \mathrm{i}\sin\theta)^{-n} = \cos(-n\theta) + \mathrm{i}\sin(-n\theta) = \cos n\theta - \mathrm{i}\sin n\theta$ both	M1
	Adding $z^n + \dfrac{1}{z^n} = 2\cos n\theta$ ∗ cso	A1 **(2)**
	(b) $\left(z + \dfrac{1}{z}\right)^6 = z^6 + 6z^4 + 15z^2 + 20 + 15z^{-2} + 6z^{-4} + z^{-6}$	M1
	$= z^6 + z^{-6} + 6(z^4 + z^{-4}) + 15(z^2 + z^{-2}) + 20$	M1
	$64\cos^6\theta = 2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20$	M1
	$32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$	A1, A1
	$(p = 1, q = 6, r = 15, s = 10)$ A1 any two correct	**(5)**
	(c) $\int \cos^6\theta \, \mathrm{d}\theta = \left(\dfrac{1}{32}\right)\int (\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10)\,\mathrm{d}\theta$	
	$= \left(\dfrac{1}{32}\right)\left[\dfrac{\sin 6\theta}{6} + \dfrac{6\sin 4\theta}{4} + \dfrac{15\sin 2\theta}{2} + 10\theta\right]$	M1 A1ft
	$[\ ....\ ]_0^{\frac{\pi}{3}} = \dfrac{1}{32}\left[-\dfrac{3}{2}\times\dfrac{\sqrt{3}}{2} + \dfrac{15}{2}\times\dfrac{\sqrt{3}}{2} + \dfrac{10\pi}{3}\right] = \dfrac{5\pi}{48} + \dfrac{3\sqrt{3}}{32}$ or exact equivalent	M1 A1 **(4)**
		[11]

Question Number	Scheme	Marks
5.	$n=1$: $1^2=\frac{1}{3}\times1\times1\times3$	B1
	(Hence result is true for $n=1$.)	
	$\sum_{r=1}^{k+1}(2r-1)^2=\sum_{r=1}^{k}(2r-1)^2+(2k+1)^2$	
	$=\frac{1}{3}k(2k-1)(2k+1)+(2k+1)^2$, by induction hypothesis	M1
	$=\frac{1}{3}(2k+1)(2k^2-k+6k+3)$	
	$=\frac{1}{3}(2k+1)(2k^2+5k+3)$	
	$=\frac{1}{3}(2k+1)(2k+3)(k+1)$	M1 A1
	$=\frac{1}{3}(k+1)[2(k+1)-1][2(k+1)+1]$	
	(Hence, if result is true for $n=k$, then it is true for $n=k+1$.)	
	By Mathematical Induction, above implies the result is true for all $n\in\mathbb{Z}^+$. ∗	
	cso	A1 **(5)**
		[5]
6.	(a) $\mathrm{f}(k+1)-\mathrm{f}(k)=3^{4k+4}+2^{4k+6}-3^{4k}-2^{4k+2}$	
	$=3^{4k}(3^4-1)+2^{4k+2}(2^4-1)$	M1
	$=3^{4k}\times80+2^{4k+2}\times15$ can be implied	A1
	$=3^{4k-1}\times240+2^{4k+2}\times15=15(16\times3^{4k-1}+2^{4k+2})$	M1
	Hence $15\mid\mathrm{f}(k+1)-\mathrm{f}(k)$ ∗ cso	A1 **(4)**
	Note: $\mathrm{f}(k+1)-\mathrm{f}(k)$ is divisible by 240 and other appropriate multiples of 15 lead to the required result.	
	(b) $n=1$: $\mathrm{f}(1)=3^4+2^6=145=5\times29 \Rightarrow 5\mid\mathrm{f}(1)$	B1
	(Hence result is true for $n=1$.)	
	From (a) $\mathrm{f}(k+1)-\mathrm{f}(k)=15\lambda$, say. By induction hypothesis $\mathrm{f}(k)=5\mu$, say.	
	$\mathrm{f}(k+1)=\mathrm{f}(k)+15\lambda=5(\mu+3\lambda) \Rightarrow 5\mid\mathrm{f}(k+1)$	M1
	(Hence, if result is true for $n=k$, then it is true for $n=k+1$.)	
	By Mathematical Induction, above implies the result is true for all $n\in\mathbb{Z}^+$. ∗	
	Accept equivalent arguments cso	A1 **(3)**
	(c) $\mathrm{f}(1)=145=5\times29$ is not divisible by 15, so result is not true for all $\mathbb{Z}^+$.	B1 **(1)**
	Note: There is no integer for which $\mathrm{f}(n)$ is divisible by 15 and any specific example should be accepted.	**[8]**

Question Number	Scheme	Marks
7.	(a) $\overrightarrow{AB} = -\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}, \quad \overrightarrow{AC} = 4\mathbf{j} + 2\mathbf{k}$ any two	B1
	$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 3 & 3 \\ 0 & 4 & 2 \end{vmatrix} = -6\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$	M1 A1 A1
	Give A1 for any two components correct or the negative of the correct answer.	**(4)**
	(b) Cartesian equation has form $3x - y + 2z = p$	
	$(2, -1, 0) \Rightarrow 6 + 1 = p$ or use of another point	M1
	$3x - y + 2z = 7$ ∗ or any multiple	A1 **(2)**
	(c) Parametric form of line is $\mathbf{r} = \begin{pmatrix} 5 \\ 5 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$ or equivalent form	M1 A1
	Substituting into equation of plane	
	$3(5 + 2\lambda) - (5 - \lambda) + 2(3 - 2\lambda) = 7$	M1
	Leading to $\lambda = -3$	A1
	$T : (-1, 8, 9)$	A1 **(5)**
	(d) $\overrightarrow{AT} = -3\mathbf{i} + 9\mathbf{j} + 9\mathbf{k}, \quad \overrightarrow{BT} = -2\mathbf{i} + 6\mathbf{j} + 6\mathbf{k}$ both	M1
	These are parallel and hence A, B and T are collinear ∗ (by the axiom of parallels)	M1 A1 **(3)** **[14]**
	Alternative to (d)	
	The equation of AB: $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + \mu(-\mathbf{i} + 3\mathbf{j} + 3\mathbf{k})$ or equivalent	
	$\mathbf{i}$: $-1 = 2 - \mu \Rightarrow \mu = 3$	M1
	$\mu = 3 \Rightarrow \overrightarrow{OT} = -\mathbf{i} + 8\mathbf{j} + 9\mathbf{k}$	M1
	Hence A, B and T are collinear ∗ cso	A1 **(3)**
	Note: Column vectors or bold-faced vectors may be used at any stage.	

Question Number	Scheme	Marks
8.	(a) Let $z=\lambda+\lambda\mathrm{i}$; $\quad w=\dfrac{\lambda+(\lambda+1)\mathrm{i}}{\lambda(1+\mathrm{i})}$	M1
	$=\dfrac{\lambda+(\lambda+1)\mathrm{i}}{\lambda(1+\mathrm{i})}\times\dfrac{1-\mathrm{i}}{1-\mathrm{i}}$	M1
	$u+\mathrm{i}v=\dfrac{(2\lambda+1)+\mathrm{i}}{2\lambda}$	A1
	$u=1+\dfrac{1}{2\lambda},\quad v=\dfrac{1}{2\lambda}$	M1
	Eliminating λ gives a line with equation $v=u-1$ or equivalent	A1 **(5)**
	(b) Let $z=\lambda-(\lambda+1)\mathrm{i}$; $\quad w=\dfrac{\lambda-\lambda\mathrm{i}}{\lambda-(\lambda+1)\mathrm{i}}$	M1
	$=\dfrac{\lambda-\lambda\mathrm{i}}{\lambda-(\lambda+1)\mathrm{i}}\times\dfrac{\lambda+(\lambda+1)\mathrm{i}}{\lambda+(\lambda+1)\mathrm{i}}$	M1
	$u+\mathrm{i}v=\dfrac{\lambda(2\lambda+1)+\lambda\mathrm{i}}{2\lambda^2+2\lambda+1}$	A1
	$u=\dfrac{\lambda(2\lambda+1)}{2\lambda^2+2\lambda+1},\quad v=\dfrac{\lambda}{2\lambda^2+2\lambda+1}$	M1
	$\dfrac{u}{v}=2\lambda+1$	
	$v=\dfrac{2\lambda}{4\lambda^2+4\lambda+2}=\dfrac{(2\lambda+1)-1}{(2\lambda+1)^2+1}=\dfrac{\frac{u}{v}-1}{\left(\frac{u}{v}\right)^2+1}$	M1
	Reducing to the circle with equation $u^2+v^2-u+v=0$ ✱ cso	M1 A1 **(7)**
	(c) 	
	ft their line	B1ft
	Circle through origin, centre in correct quadrant	B1
	Intersections correctly placed	B1 **(3)**
		[15]

Question Number	Scheme	Marks
8.	Alternative for (b)	
	Let $z=\lambda-(\lambda+1)\mathrm{i}$; $\quad w=\dfrac{\lambda-\lambda\mathrm{i}}{\lambda-(\lambda+1)\mathrm{i}}$	M1
	$=\dfrac{\lambda-\lambda\mathrm{i}}{\lambda-(\lambda+1)\mathrm{i}}\times\dfrac{\lambda+(\lambda+1)\mathrm{i}}{\lambda+(\lambda+1)\mathrm{i}}$	M1
	$u+\mathrm{i}v=\dfrac{\lambda(2\lambda+1)+\lambda\mathrm{i}}{2\lambda^2+2\lambda+1}$	A1
	$u=\dfrac{\lambda(2\lambda+1)}{2\lambda^2+2\lambda+1},\quad v=\dfrac{\lambda}{2\lambda^2+2\lambda+1}$	M1
	$u^2+v^2-u+v=\left(\dfrac{\lambda(2\lambda+1)}{2\lambda^2+2\lambda+1}\right)^2+\left(\dfrac{\lambda}{2\lambda^2+2\lambda+1}\right)^2-\dfrac{\lambda(2\lambda+1)}{2\lambda^2+2\lambda+1}+\dfrac{\lambda}{2\lambda^2+2\lambda+1}$	
	$=\dfrac{(4\lambda^4+4\lambda^3+\lambda^2)+\lambda^2-2\lambda^2(2\lambda^2+2\lambda+1)}{(2\lambda^2+2\lambda+1)^2}$	M1
	$=0$ ∗	M1 A1 **(7)**
8.	Alternative for (b)	
	Let $z=\lambda-(\lambda+1)\mathrm{i}$; $\quad u+\mathrm{i}v=\dfrac{\lambda-\lambda\mathrm{i}}{\lambda-(\lambda+1)\mathrm{i}}$	M1
	$(u+\mathrm{i}v)(\lambda-(\lambda+1)\mathrm{i})=\lambda-\lambda\mathrm{i}$	M1
	$u\lambda+v(\lambda+1)+[v\lambda-u(\lambda+1)]\mathrm{i}=\lambda-\lambda\mathrm{i}$	A1
	Equating real & imaginary parts	
	$u\lambda+v(\lambda+1)=\lambda$ (i) $\qquad v\lambda-\lambda u-u=-\lambda$ (ii)	M1
	From (i) $\lambda=\dfrac{v}{1-u-v}$ $\qquad$ From (ii) $\lambda=\dfrac{u}{1-u+v}$	
	$\dfrac{v}{1-u-v}=\dfrac{u}{1-u+v}$	M1
	Reducing to the circle with equation $u^2+v^2-u+v=0$ ∗	M1 A1 **(7)**