GCE Examinations

Further Pure Mathematics Module FP2

Advanced Subsidiary / Advanced Level

Paper H

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Rosemary Smith & Shaun Armstrong

1. A curve has the equation

$$2x^2 + y^2 = 4 .$$

Find the radius of curvature of the curve at the point $(1, -\sqrt{2})$. **(8 marks)**

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2. *(a)* Using the definition of $\cosh x$ in terms of exponential functions show that $\cosh x$ is an even function.

(2 marks)

(b) Given that $x > 0$ and $y > 0$, solve the simultaneous equations

$$\ln(xy) = \text{arcosh}\left(\frac{5}{3}\right),$$

$$\cosh(3x - y) = 1.$$

(6 marks)

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3. Find

$$\int \frac{1}{13\cosh x - 5\sinh x}\ \mathrm{d}x .$$

(8 marks)

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4. *(a)* Given that $y = \arcsin(2x - 1)$, prove that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{x - x^2}} .$$

(4 marks)

The tangent to the curve $y = \arcsin(2x - 1)$ at the point where $x = \frac{3}{4}$ meets the y-axis at A.

(b) Find the exact value of the y-coordinate of A. **(5 marks)**

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5. The point $P\ (at^2, 2at)$, $t \neq 0$, lies on the parabola C with equation $y^2 = 4ax$.

(a) Show that an equation of the tangent to C at P is

$$yt = x + at^2 .$$ **(4 marks)**

The tangent to C at P meets the x-axis at Q and the y-axis at R.

M is the mid-point of QR.

(b) Find the coordinates of M. **(3 marks)**

Given that OM is perpendicular to OP, where O is the origin,

(c) show that $t^2 = 2$. **(4 marks)**

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6. $$I_n = \int \frac{\cos n\theta}{\sin\theta}\ \mathrm{d}\theta, \quad n \in \mathbb{N}.$$

(a) By considering $I_n - I_{n-2}$, or otherwise, show that

$$I_n = \frac{2\cos(n-1)\theta}{n-1} + I_{n-2} .$$ **(5 marks)**

(b) Hence evaluate

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos 5\theta}{\sin\theta}\ \mathrm{d}\theta ,$$

leaving your answer in terms of natural logarithms. **(8 marks)**

Turn over

7. The ellipse C has equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, where a and b are positive constants and $a > b$.

The coordinates of the foci of C are $(\pm\sqrt{3}, 0)$, and the equations of its directrices are $x = \pm\dfrac{4}{\sqrt{3}}$.

(a) Find the value of a and the value of b. **(4 marks)**

The ellipse is rotated completely about the x-axis.

(b) Show that the area of the surface of revolution generated is given by

$$A = \frac{\pi}{2}\int_{-2}^{2} \sqrt{16 - 3x^2}\ \mathrm{d}x.$$

(6 marks)

(c) Use integration to show that

$$A = \tfrac{8}{9}\pi^2\sqrt{3} + 2\pi.$$

(8 marks)

END