

GCE Examinations

Further Pure Mathematics

Module FP2

Advanced Subsidiary / Advanced Level

Paper G

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and/or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.



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1. Given that $y = e^{\arctan x}$,

(a) find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. **(4 marks)**

The curve $y = e^{\arctan x}$ has a point of inflexion.

(b) Find the coordinates of this point of inflexion. **(3 marks)**

2. (a) Prove that

$$\frac{d}{dx}(\operatorname{arcosh} x) = \frac{1}{\sqrt{x^2 - 1}}. \quad \text{(3 marks)}$$

(b) Find

$$\int \operatorname{arcosh} x \, dx. \quad \text{(4 marks)}$$

3. Find

$$\int_0^{\frac{\pi}{4}} \frac{1}{1 + \sin 2x} \, dx. \quad \text{(8 marks)}$$

4. (a) Find

$$\int \frac{1}{\sqrt{4x^2 - 4x + 10}} dx. \quad (6 \text{ marks})$$

- (b) Hence evaluate

$$\int_{\frac{1}{2}}^2 \frac{1}{\sqrt{4x^2 - 4x + 10}} dx,$$

giving your answer in terms of natural logarithms. (3 marks)

5. (a) On the same axes sketch the curves with equations $y = 2 - \tanh x$ and $y = 3 \operatorname{sech} x$, giving the coordinates of the points of intersection of the curves with the coordinate axes and the equations of the asymptotes. (5 marks)

- (b) Solve the equation

$$2 - \tanh x = 3 \operatorname{sech} x,$$

giving your answers to 2 decimal places. (7 marks)

6.
$$I_n = \int_0^{\frac{\pi}{2}} \sin^n x \, dx, \quad n \geq 0.$$

- (a) Show that

$$I_n = \frac{n-1}{n} I_{n-2}, \quad n \geq 2. \quad (7 \text{ marks})$$

The curve C is defined by $y = \sin^2 x$, $0 \leq x \leq \pi$.

The area bounded by C and the positive x -axis is rotated through 2π radians about the x -axis.

- (b) Find the volume of the solid generated giving your answer in terms of π . (7 marks)
-

Turn over

7.

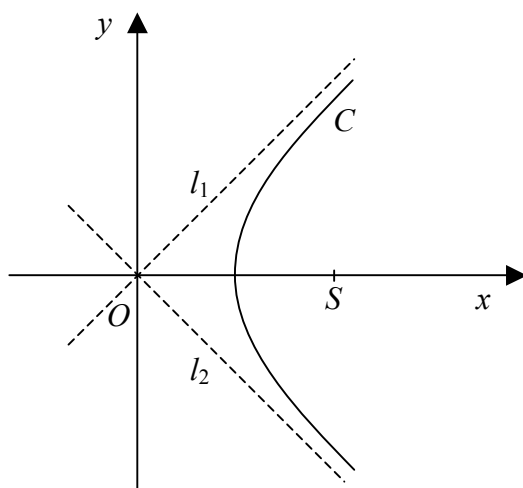


Fig. 1

Figure 1 shows the curve C which is part of the hyperbola with parametric equations

$$x = a \cosh t, \quad y = 2a \sinh t,$$

where a is a positive constant and $x \geq a$. The lines l_1 and l_2 are asymptotes to C .

(a) Show that the radius of curvature of C at its vertex is $4a$. (6 marks)

(b) Show that an equation of the tangent to C at the point $P(a \cosh p, 2a \sinh p)$ is

$$2x \cosh p - y \sinh p = 2a. \quad \text{(4 marks)}$$

The tangent to the curve C at P meets the asymptote l_1 at Q .

Given that QS is parallel to the y -axis, where S is the focus,

(c) show that $p = \frac{1}{2} \ln 5$. (8 marks)

END