

GCE Examinations

Further Pure Mathematics

Module FP2

Advanced Subsidiary / Advanced Level

Paper F

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and/or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 8 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.



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1. $f(x) = \operatorname{artanh}(\sin x)$.

Show that $f'(x) = \sec x$. **(4 marks)**

2. Find the length of the arc of the curve with equation $y = \ln(\sec x)$ between $x = 0$ and $x = \frac{\pi}{3}$, giving your answer in terms of natural logarithms.

(7 marks)

3. A curve has parametric equations

$$x = t^2, \quad y = t^3.$$

Show that the radius of curvature of the curve at the point $(1, 1)$ is $\frac{13\sqrt{13}}{6}$. **(7 marks)**

4. $I_n = \int_1^e (\ln x)^n dx$.

(a) Prove that, for $n \in \mathbb{Z}^+$,

$$I_n = e - nI_{n-1}. \quad \text{span style="float: right;">**(4 marks)**$$

(b) Find I_3 , leaving your answer in terms of e . **(5 marks)**

5.

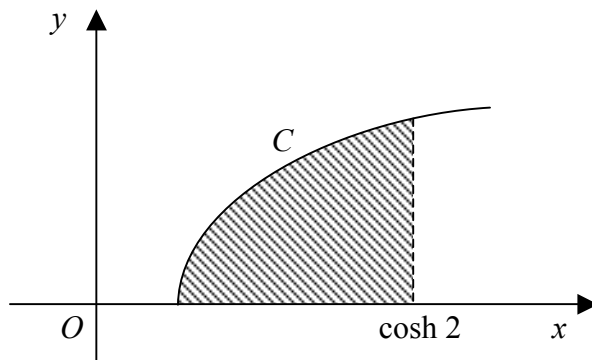


Fig. 1

Figure 1 shows the curve C which has equation $y = \operatorname{arcosh} x$.

The shaded region bounded by C , the x -axis and the line $x = \cosh 2$ is rotated through 2π about the y -axis.

The volume of revolution of the solid generated is $a\pi$.

Find the value of a to one decimal place.

(10 marks)

6.

$$f(x) \equiv \frac{3x-7}{(x+1)(x^2+4)}, \quad x \neq -1.$$

(a) Express $f(x)$ in partial fractions.

(4 marks)

(b) Show that

$$\int_0^2 f(x) \, dx = \frac{\pi}{8} + \ln\left(\frac{2}{9}\right).$$

(7 marks)

Turn over

7. The ellipse C has equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a and b are positive constants and $a > b$.

(a) Find an equation of the normal to C at the point $P (a \cos \theta, b \sin \theta)$. **(5 marks)**

The normal to C at P meets the x -axis at Q .

R is the foot of the perpendicular from P to the x -axis.

(b) Show that $\frac{OQ}{OR} = e^2$, where e is the eccentricity of C . **(7 marks)**

8. (a) Using the definitions of hyperbolic functions in terms of exponential functions prove that

$$\operatorname{arsinh} x = \ln(x + \sqrt{x^2 + 1}). \quad \textbf{(6 marks)}$$

(b) On the same axes sketch the graphs of $y = \sinh x$ and $y = \operatorname{arsinh} x$. **(3 marks)**

(c) Solve the equation

$$x = \sinh[\ln(3x - 2)], \quad x > \frac{2}{3}. \quad \textbf{(6 marks)}$$

END