

GCE Examinations

Further Pure Mathematics Module FP2

Advanced Subsidiary / Advanced Level

Paper B

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and/or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working will gain no credit.



Written by Rosemary Smith & Shaun Armstrong

© Solomon Press

These sheets may be copied for use solely by the purchaser's institute.

1. Given that

$$y \arccos x - \frac{x}{\pi} e^{2x} - 1 = 0,$$

find the value of $\frac{dy}{dx}$ at the point where $x = 0$, giving your answer in terms of π . **(7 marks)**

2. $f(x) = 5 \cosh x + 3 \sinh x$.

The minimum value of $f(x)$ occurs at the point $(p \ln q, r)$ where p , q and r are integers.

Find the values of p , q and r . **(8 marks)**

3. The line $y = mx + c$ is a tangent to the rectangular hyperbola with equation $xy = -9$.

(a) Show that $c = \pm 6\sqrt{m}$. **(4 marks)**

(b) Hence, or otherwise, find the equations of the tangents from the point $(4, -2)$ to the rectangular hyperbola $xy = -9$.

(5 marks)

4. The curve C is defined by

$$y^2 = x, \quad x \geq 0, \quad y \geq 0.$$

The region between C , the x -axis and the line $x = 1$ is rotated through 2π about the x -axis.

Show that the area of the surface generated is

$$\frac{\pi}{6}(5\sqrt{5} - 1). \quad \textbf{(11 marks)}$$

5. (a) Using the definition of $\cosh x$ in terms of exponential functions, express $\operatorname{sech} x$ in terms of e^x and e^{-x} . (1 mark)
- (b) Sketch the graph of $y = \operatorname{sech} x$. (2 marks)
- (c) Show that $\int \operatorname{sech} x \, dx = 2 \arctan e^x + c$. (4 marks)

The curve C has equation $y = \operatorname{sech} x$. The region between C , the x -axis and the lines $x = -a$ and $x = a$, where a is a positive constant, is rotated through 2π about the x -axis.

- (d) Find the volume of revolution of the solid generated. (4 marks)
- (e) Find the limit of the volume of revolution as $a \rightarrow \infty$. (1 mark)
-

6.
$$I_n = \int_0^{\sqrt{2}} (2-x^2)^n \, dx, \quad n \geq 0.$$

- (a) Show that

$$I_n = \frac{4n}{2n+1} I_{n-1}, \quad n \geq 1. \quad \text{(9 marks)}$$

- (b) Hence evaluate I_3 , leaving your answer in surd form. (4 marks)
-

7. The curve C has intrinsic equation

$$s = \ln(\tan \frac{1}{2} \psi), \quad 0 < \psi \leq \frac{\pi}{2}.$$

- (a) Show that radius of curvature of C is given by $\rho = \operatorname{cosec} \psi$. (4 marks)

Given that $y = \psi = \frac{\pi}{2}$ when $x = 0$,

- (b) show that $y = \psi$, (4 marks)
- (c) use integration to show that a Cartesian equation of C is $x = \ln(\sin y)$. (7 marks)

END