GCE Examinations
Advanced Subsidiary / Advanced Level

# Further Pure Mathematics
# Module FP2

Paper D

# MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

*Written by Rosemary Smith & Shaun Armstrong*

# FP2 Paper D – Marking Guide

| | | | | |
|---|---|---|---|---|
| **1.** | *(a)* | $\dfrac{dy}{dx} = \dfrac{-(x^2+1)\text{cosech}\, x \coth x - 2x \text{ cosech}\, x}{(x^2+1)^2}$ | M2 A2 | |
| | *(b)* | $x = 0.5,\ \dfrac{dy}{dx} = -4.55$ (2dp) | A1 | **(5)** |

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| **2.** | $\rho = \dfrac{ds}{d\psi} = 2(s+a)\ \therefore \int \frac{1}{s+a}\ ds = \int 2\ d\psi$ | M1 A1 | |
| | $\ln\lvert s+a \rvert = 2\psi + c$ | A1 | |
| | $s + a = e^{2\psi + c} = e^{2\psi} \times e^{c}$ | M1 | |
| | $\therefore\ s = Ae^{2\psi} - a$ [where $A = e^{c}$] | A1 | **(5)** |

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| **3.** | *(a)* | $\sinh 3x \equiv \sinh(2x + x)$ | M1 | |
| | | $\equiv \sinh 2x \cosh x + \cosh 2x \sinh x$ | A1 | |
| | | $\equiv 2 \sinh x \cosh^2 x + (1 + 2\sinh^2 x)\sinh x$ | M1 | |
| | | $\equiv 2 \sinh x(1 + \sinh^2 x) + \sinh x + 2\sinh^3 x$ | M1 | |
| | | $\equiv 2 \sinh x + 2\sinh^3 x + \sinh x + 2\sinh^3 x$ | | |
| | | $\equiv 4\sinh^3 x + 3\sinh x$ | A1 | |
| | *(b)* | $4\sinh^3 x + 3\sinh x = 7\sinh^2 x$ | | |
| | | $\sinh x(4\sinh^2 x - 7\sinh x + 3) = 0$ | M1 | |
| | | $\sinh x(4\sinh x - 3)(\sinh x - 1) = 0$ | M1 | |
| | | $\sinh x = 0$ or $\frac{3}{4}$ or $1$ | A1 | |
| | | $x = 0$ or $\ln\left(\frac{3}{4} + \sqrt{1 + \frac{9}{16}}\right)$ or $\ln\left(1 + \sqrt{1+1}\right)$ | M1 | |
| | | $x = 0$ or $\ln 2$ or $\ln(1 + \sqrt{2})$ | A2 | **(11)** |

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**4.** *(a)* $\int \frac{1}{\sqrt{9-4x^2}}\,dx = \int \frac{1}{2}\frac{1}{\sqrt{\frac{9}{4}-x^2}}\,dx$ M1

$= \frac{1}{2}\arcsin\left(\frac{2x}{3}\right) + c$ M1 A1

*(b)* $\int \frac{1-2x}{\sqrt{9-4x^2}}\,dx = \frac{1}{2}\arcsin\left(\frac{2x}{3}\right) - \int \frac{2x}{\sqrt{9-4x^2}}\,dx$ M1

$= \frac{1}{2}\arcsin\left(\frac{2x}{3}\right) + \frac{1}{2}\sqrt{9-4x^2} + c$ M1 A1

*(c)* $\int_1^y \frac{1}{y}\,dy = \int_0^x \frac{1-2x}{\sqrt{9-4x^2}}\,dx$ M1 A1

$[\ln|y|]_1^y = [\frac{1}{2}\arcsin\left(\frac{2x}{3}\right) + \frac{1}{2}\sqrt{9-4x^2}\,]_0^x$ A1

$\ln|y| - \ln 1 = \frac{1}{2}\arcsin\left(\frac{2x}{3}\right) + \frac{1}{2}\sqrt{9-4x^2} - (\frac{1}{2}\arcsin 0 + \frac{3}{2})$ M1 A1

$\ln|y| = \frac{1}{2}\arcsin\left(\frac{2x}{3}\right) + \frac{1}{2}\sqrt{9-4x^2} - \frac{3}{2}$ A1 **(12)**

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**5.** *(a)* $2y\frac{dy}{dx} = 4a \;\therefore\; \frac{dy}{dx} = \frac{2a}{y}$ M1

at $P$, $\frac{dy}{dx} = \frac{2a}{2ap} = \frac{1}{p}$ A1

eqn. is $y - 2ap = \frac{1}{p}(x - ap^2)$ M1

giving $yp = x + ap^2$ A1

*(b)* grad of $PQ = \frac{2ap-2aq}{ap^2-aq^2} = \frac{2(p-q)}{(p+q)(p-q)} = \frac{2}{p+q}$ M1 A1

grad of $PS = \frac{2ap-0}{ap^2-a} = \frac{2p}{p^2-1}$ A1

$\therefore \frac{2}{p+q} = \frac{2p}{p^2-1}$ M1

$p^2 - 1 = p(p+q)$

$p^2 - 1 = p^2 + pq$

$\therefore pq = -1$ A1

*(c)* tangent at $P$: $yp = x + ap^2$ (i)

tangent at $Q$: $yq = x + aq^2$ (ii)

(i) $\times q$: $ypq = xq + ap^2q$

(ii) $\times p$: $ypp = xp + apq^2$ M1

subtracting $0 = x(p-q) + apq(q-p)$ M1

$0 = x - apq$ A1

$pq = -1 \;\therefore\; x = -a \;\therefore$ meet on directrix A1 **(13)**

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| **6.** *(a)* | $u = \sec^{n-2}x$, $u' = (n-2)\sec^{n-3}x \sec x \tan x$; $v' = \sec^2 x$, $v = \tan x$ | M1 A1 | |
| | $I_n = [\sec^{n-2}x \tan x]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} (n-2)\sec^{n-2}x \tan^2 x \ \mathrm{d}x$ | A1 | |
| | $I_n = (\sqrt{2})^{n-2} - 0 - (n-2)\int_0^{\frac{\pi}{4}} \sec^{n-2}x(\sec^2 x - 1) \ \mathrm{d}x$ | M1 A1 | |
| | $I_n = (\sqrt{2})^{n-2} - (n-2)\int_0^{\frac{\pi}{4}} \sec^n x \ \mathrm{d}x + (n-2)\int_0^{\frac{\pi}{4}} \sec^{n-2}x \ \mathrm{d}x$ | | |
| | $I_n = (\sqrt{2})^{n-2} - (n-2)I_n + (n-2)I_{n-2}$ | M1 | |
| | $(n-1)I_n = (\sqrt{2})^{n-2} + (n-2)I_{n-2}$ | A1 | |
| *(b)* | $I_1 = \int_0^{\frac{\pi}{4}} \sec x \ \mathrm{d}x = [\ln \lvert \sec x + \tan x \rvert]_0^{\frac{\pi}{4}}$ | M1 | |
| | $= \ln(\sqrt{2}+1) - \ln(1+0) = \ln(\sqrt{2}+1)$ | M1 A1 | |
| | $2I_3 = (\sqrt{2})^1 + I_1 = \sqrt{2} + \ln(\sqrt{2}+1)$ | M1 A1 | |
| | $I_3 = \frac{1}{2}\sqrt{2} + \frac{1}{2}\ln(\sqrt{2}+1)$ | A1 | **(13)** |

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| **7.** *(a)* | $x^2 = a^2\sinh^2 u$, $x = a\sinh u$, $\frac{\mathrm{d}x}{\mathrm{d}u} = a\cosh u$ | M1 A1 | |
| | $\int \sqrt{a^2+x^2} \ \mathrm{d}x = \int \sqrt{a^2 + a^2\sinh^2 u}\,(a\cosh u) \ \mathrm{d}u$ | M1 | |
| | $= \int a^2\cosh^2 u \ \mathrm{d}u$ | A1 | |
| | $= \frac{1}{2}a^2 \int \cosh 2u + 1 \ \mathrm{d}u$ | M1 | |
| | $= \frac{1}{2}a^2[\frac{1}{2}\sinh 2u + u] + c$ | A1 | |
| | $= \frac{1}{2}a^2\sinh u\cosh u + \frac{1}{2}a^2 u + c$ | M1 | |
| | $= \frac{1}{2}a^2 \times \frac{x}{a} \times \sqrt{1+\frac{x^2}{a^2}} + \frac{1}{2}a^2\,\mathrm{arsinh}(\frac{x}{a}) + c$ | M1 | |
| | $= \frac{1}{2}ax\sqrt{1+\frac{x^2}{a^2}} + \frac{1}{2}a^2\,\mathrm{arsinh}(\frac{x}{a}) + c$ | | |
| | $= \frac{1}{2}x\sqrt{a^2+x^2} + \frac{1}{2}a^2\,\mathrm{arsinh}(\frac{x}{a}) + c$ | A1 | |
| *(b)* | $x = 2t$, $\frac{\mathrm{d}x}{\mathrm{d}t} = 2$; $y = t^2$, $\frac{\mathrm{d}y}{\mathrm{d}t} = 2t$ | M1 | |
| | $s = \int_0^3 \sqrt{4+4t^2} \ \mathrm{d}t$ | M1 A1 | |
| | $s = 2\int_0^3 \sqrt{1+t^2} \ \mathrm{d}t$ | A1 | |
| *(c)* | $s = 2[\frac{t}{2}\sqrt{1+t^2} + \frac{1}{2}\mathrm{arsinh}\, t]_0^3$ | M1 | |
| | $s = 2[(\frac{3}{2}\sqrt{10} + \frac{1}{2}\mathrm{arsinh}\, 3) - (0+0)] = 3\sqrt{10} + \mathrm{arsinh}\, 3$ | M1 A1 | **(16)** |

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Total **(75)**

# Performance Record – FP2 Paper D

| Question no. | **1** | **2** | **3** | **4** | **5** | **6** | **7** | Total |
|---|---|---|---|---|---|---|---|---|
| Topic(s) | diff. hyp. fns | intrinsic coords | eqn. in hyp. fns. | integr. std. forms | parabola, tangent | reduction formula | integr. using hyp. sub., arc length | |
| Marks | 5 | 5 | 11 | 12 | 13 | 13 | 16 | 75 |
| Student | | | | | | | | |
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