GCE Examinations

Advanced Subsidiary / Advanced Level

Further Pure Mathematics
Module FP2

Paper C

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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FP2 Paper C – Marking Guide

1.	$\rho = \dfrac{ds}{d\psi} = 12\sec^2\psi \times \sec\psi\tan\psi$		M1 A1	
	$= 12\sec^3\psi\tan\psi$		A1	
	$\psi = \frac{\pi}{4}$, $\rho = 12(\sqrt{2})^3(1) = 24\sqrt{2}$		M1 A1	**(5)**

2.	$\dfrac{5\cosh x}{\sinh x} + 1 = \dfrac{7}{\sinh x}$	M1	
	$5\cosh x + \sinh x = 7$	A1	
	$\frac{5}{2}(e^x + e^{-x}) + \frac{1}{2}(e^x - e^{-x}) = 7$	M1	
	$3e^x + 2e^{-x} = 7$		
	$3e^{2x} - 7e^x + 2 = 0$	A1	
	$(3e^x - 1)(e^x - 2) = 0$	M1	
	$e^x = \frac{1}{3}$ or 2 $\therefore x = \ln\frac{1}{3}$ or $\ln 2$	M1 A1	**(7)**

3.	*(a)*	let $y = \arccos x$ $\therefore \cos y = x$		
		$\therefore -\sin y\dfrac{dy}{dx} = 1$	M1	
		$\dfrac{dy}{dx} = \dfrac{-1}{\sqrt{1-\cos^2 y}} = \dfrac{-1}{\sqrt{1-x^2}}$	M1 A1	
	(b)	$\dfrac{dy}{dx} = \dfrac{-1}{\sqrt{1-x^2}} - \dfrac{1}{2}\,\dfrac{-2x}{1-x^2} = \dfrac{-1}{\sqrt{1-x^2}} + \dfrac{x}{1-x^2}$	M1 A1	
		S.P. $\therefore \dfrac{dy}{dx} = 0$ $\therefore \dfrac{x}{1-x^2} = \dfrac{1}{\sqrt{1-x^2}}$	M1	
		$x = \sqrt{1-x^2}$		
		$x^2 = 1 - x^2$	M1	
		$x^2 = \frac{1}{2}$, $0 < x < 1$, $\therefore x = \frac{1}{\sqrt{2}}$	A1	
		$x = \frac{1}{\sqrt{2}}$, $y = \frac{\pi}{4} - \frac{1}{2}\ln\frac{1}{2}$ $\therefore (\frac{1}{\sqrt{2}}, \frac{\pi}{4} - \frac{1}{2}\ln\frac{1}{2})$	M1 A1	**(10)**

4. *(a)* $3 - 6x - 9x^2 \equiv 3 - [(3x + 1)^2 - 1]$ M1

$\equiv 4 - (3x + 1)^2 \quad \therefore\ a = 4,\ b = 3,\ c = 1$ A1

(b) $\displaystyle\int \frac{1}{\sqrt{3-6x-9x^2}}\,\mathrm{d}x = \int \frac{1}{\sqrt{4-(3x+1)^2}}\,\mathrm{d}x$

$u = 3x + 1,\ \dfrac{\mathrm{d}u}{\mathrm{d}x} = 3$ M1

$= \displaystyle\int \frac{1}{3}\,\frac{1}{\sqrt{4-u^2}}\,\mathrm{d}u$ A1

$= \frac{1}{3}\arcsin\left(\frac{u}{2}\right) + c = \frac{1}{3}\arcsin\left(\frac{3x+1}{2}\right) + c$ M1 A1

(c) $\displaystyle\int_{-\frac{1}{3}}^{0} \frac{1}{3-6x-9x^2}\,\mathrm{d}x = \int_{-\frac{1}{3}}^{0} \frac{1}{4-(3x+1)^2}\,\mathrm{d}x$

$u = 3x + 1,\ \dfrac{\mathrm{d}u}{\mathrm{d}x} = 3$ M1

$= \displaystyle\int_{0}^{1} \frac{1}{3}\,\frac{1}{4-u^2}\,\mathrm{d}u$ A1

$= \frac{1}{3}\left[\frac{1}{2}\,\text{artanh}\left(\frac{u}{2}\right)\right]_0^1$ M1 A1

$= \frac{1}{6}\left[\text{artanh}\,\frac{1}{2} - \text{artanh}\,0\right]$

$= \frac{1}{6}\left[\frac{1}{2}\ln\left(\dfrac{1+\frac{1}{2}}{1-\frac{1}{2}}\right) - 0\right] = \frac{1}{12}\ln 3$ M1 A1 **(12)**

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5. *(a)* $\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{\mathrm{e}^x - \mathrm{e}^{-x}}{\mathrm{e}^x + \mathrm{e}^{-x}}$ B1

(b) let $y = \text{artanh}\left(\frac{x^2-1}{x^2+1}\right) \quad \therefore\ \tanh y = \dfrac{\mathrm{e}^y - \mathrm{e}^{-y}}{\mathrm{e}^y + \mathrm{e}^{-y}} = \dfrac{x^2-1}{x^2+1}$ M1 A1

$(\mathrm{e}^y - \mathrm{e}^{-y})(x^2 + 1) = (\mathrm{e}^y + \mathrm{e}^{-y})(x^2 - 1)$ M1

$\mathrm{e}^y[(x^2 + 1) - (x^2 - 1)] = \mathrm{e}^{-y}[(x^2 - 1) + (x^2 + 1)]$ A1

$2\mathrm{e}^y = 2x^2\mathrm{e}^{-y}$

$\mathrm{e}^{2y} = x^2$ M1

$2y = \ln x^2 = 2\ln x \quad \therefore\ y = \mathrm{f}(x) = \ln x$ A1

(c) $\displaystyle\int_{1}^{2\mathrm{e}} \text{artanh}\left(\frac{x^2-1}{x^2+1}\right)\mathrm{d}x = \int_{1}^{2\mathrm{e}} \ln x\ \mathrm{d}x$

$u = \ln x,\ u' = \frac{1}{x};\quad v' = 1,\ v = x$ M1

$I = [x\ln x]_1^{2\mathrm{e}} - \displaystyle\int_{1}^{2\mathrm{e}} x \times \frac{1}{x}\,\mathrm{d}x$ A1

$= [x\ln x - x]_1^{2\mathrm{e}}$ A1

$= 2\mathrm{e}\ln(2\mathrm{e}) - 2\mathrm{e} - (\ln 1 - 1)$

$= 2\mathrm{e}(\ln 2 + \ln \mathrm{e}) - 2\mathrm{e} + 1$ M1

$= 2\mathrm{e}\ln 2 + 2\mathrm{e} - 2\mathrm{e} + 1 = 2\mathrm{e}\ln 2 + 1$ A1 **(12)**

6.	*(a)*	$\frac{2x}{25} + \frac{2y}{9}\frac{dy}{dx} = 0 \;\therefore\; \frac{dy}{dx} = -\frac{9x}{25y}$	M1 A1	
		at P $\frac{dy}{dx} = -\frac{9\times 5\cos\theta}{25\times 3\sin\theta} = -\frac{3\cos\theta}{5\sin\theta}$	M1	
		$\therefore$ eqn. of normal is		
		$y - 3\sin\theta = \frac{5\sin\theta}{3\cos\theta}(x - 5\cos\theta)$	M1 A1	
		or $5x\sin\theta - 3y\cos\theta = 16\sin\theta\cos\theta$		
	(b)	at Q, $y = 0$, $x = \frac{16}{5}\cos\theta \;\therefore\; Q(\frac{16}{5}\cos\theta, 0)$	M1 A1	
		at R, $x = 0$, $y = -\frac{16}{3}\sin\theta \;\therefore\; R(0, -\frac{16}{3}\sin\theta)$	M1 A1	
		$\therefore$ S is $(\frac{16}{5}\cos\theta, -\frac{16}{3}\sin\theta)$	M1	
		of form $(a\cos\theta, b\sin\theta)$ $\therefore$ ellipse	A1	
		$\cos\theta = \frac{5}{16}x$, $\sin\theta = -\frac{3}{16}y$		
		using $\cos^2\theta + \sin^2\theta = 1$ gives $\frac{25x^2}{256} + \frac{9y^2}{256} = 1$	M1 A1	**(13)**

7.	*(a)*	$u = \cos^{n-1}2t$, $u' = 2(n-1)\cos^{n-2}2t(-\sin 2t)$; $v' = \cos 2t$, $v = \frac{1}{2}\sin 2t$	M1	
		$I_n(x) = [\frac{1}{2}\cos^{n-1}2t\sin 2t]_0^x - \int_0^x -(n-1)\cos^{n-2}2t\sin^2 2t \; dt$	A1	
		$I_n(x) = \frac{1}{2}\cos^{n-1}2x\sin 2x - 0 + (n-1)\int_0^x \cos^{n-2}2t(1-\cos^2 2t) \; dt$	M1 A1	
		$I_n(x) = \frac{1}{2}\cos^{n-1}2x\sin 2x + (n-1)\int_0^x \cos^{n-2}2t \, dt - (n-1)\int_0^x \cos^n 2t \, dt$	A1	
		$I_n(x) = \frac{1}{2}\cos^{n-1}2x\sin 2x + (n-1)I_{n-2}(x) - (n-1)I_n(x)$	M1	
		$\therefore nI_n(x) = \frac{1}{2}\sin 2x\cos^{n-1}2x + (n-1)I_{n-2}(x)$	A1	
	(b)	$I_0(\frac{\pi}{4}) = \int_0^{\frac{\pi}{4}} dt = [t]_0^{\frac{\pi}{4}} = \frac{\pi}{4}$	M1 A1	
	(c)	area $= \frac{1}{2}\int_0^{\frac{\pi}{4}} a^2\cos^4 2\theta \; d\theta = \frac{1}{2}a^2 I_4(\frac{\pi}{4})$	M1 A1	
		$nI_n(\frac{\pi}{4}) = \frac{1}{2}\sin\frac{\pi}{2}\cos^{n-1}(\frac{\pi}{2}) + (n-1)I_{n-2}(\frac{\pi}{4})$	M1	
		$\therefore I_n(\frac{\pi}{4}) = \frac{n-1}{n}I_{n-2}(\frac{\pi}{4})$	A1	
		$I_2(\frac{\pi}{4}) = \frac{1}{2}I_0(\frac{\pi}{4}) = \frac{\pi}{8}$	M1	
		$I_4(\frac{\pi}{4}) = \frac{3}{4}I_2(\frac{\pi}{4}) = \frac{3\pi}{32}$	A1	
		$\therefore$ area $= \frac{1}{2}a^2 \times \frac{3\pi}{32} = \frac{3}{64}\pi a^2$	A1	**(16)**

Total **(75)**

Performance Record – FP2 Paper C

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	rad. of curv.	eqn. in hyp. fns.	diff. inv. trig.	integr. std. forms	inv. hyp. fns.	ellipse, normal, loci	reduction formula	
Marks	5	7	10	12	12	13	16	75
Student								