GCE Examinations

Advanced Subsidiary / Advanced Level

Further Pure Mathematics
Module FP2

Paper A

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

Written by Rosemary Smith & Shaun Armstrong

FP2 Paper A – Marking Guide

1.	$y = x + 2x^2 + 5x^3,\ \dfrac{dy}{dx} = 1 + 4x + 15x^2,\ \dfrac{d^2y}{dx^2} = 4 + 30x$	M1 A1
	when $x = 0,\ \dfrac{dy}{dx} = 1,\ \dfrac{d^2y}{dx^2} = 4$	A1
	$\rho = \dfrac{(1+1^2)^{\frac{3}{2}}}{4} = \dfrac{2\sqrt{2}}{4} = \dfrac{1}{\sqrt{2}}$	M1 A1 **(5)**
2.	$u = x,\ u' = 1;\ v' = \text{sech}^2 x,\ v = \tanh x$	M1
	$\int_0^{\ln 2} x\,\text{sech}^2 x\ dx = [x \tanh x]_0^{\ln 2} - \int_0^{\ln 2} \tanh x\ dx$	A1
	$= [x \tanh x - \ln(\cosh x)]_0^{\ln 2}$	M1 A1
	$= (\ln 2 \times \dfrac{e^{\ln 2} - e^{-\ln 2}}{e^{\ln 2} + e^{-\ln 2}} - \ln\left(\dfrac{e^{\ln 2} + e^{-\ln 2}}{2}\right)) - (0 - \ln 1)$	M1 A1
	$= \ln 2 \times \dfrac{2 - \frac{1}{2}}{2 + \frac{1}{2}} - \ln\left(\dfrac{2 + \frac{1}{2}}{2}\right)$	M1
	$= \frac{3}{5}\ln 2 - \ln\frac{5}{4}$	A1 **(8)**
3.	*(a)* let $y = \arcsin 2x\ \therefore\ \sin y = 2x$	
	$\therefore\ \cos y \dfrac{dy}{dx} = 2$	M1
	$\dfrac{dy}{dx} = \dfrac{2}{\sqrt{1 - \sin^2 y}} = \dfrac{2}{\sqrt{1 - 4x^2}}$	M1 A1
	(b) $f'(x) = 2x \times \dfrac{2}{\sqrt{1-4x^2}} + 2\arcsin 2x + \frac{1}{2}(-8x)(1 - 4x^2)^{-\frac{1}{2}}$	M1 A1
	$= \dfrac{4x}{\sqrt{1-4x^2}} + 2\arcsin 2x - \dfrac{4x}{\sqrt{1-4x^2}} = 2\arcsin 2x$	A1
	$f''(x) = \dfrac{4}{\sqrt{1-4x^2}}$	A1
	$f''(x)[f(x) - xf'(x)] = \dfrac{4}{\sqrt{1-4x^2}}(2x\arcsin 2x + \sqrt{1-4x^2} - 2x\arcsin 2x)$	M1
	$= \dfrac{4}{\sqrt{1-4x^2}} \times \sqrt{1-4x^2} = 4$	A1 **(9)**

4.	*(a)*	at A, $y = 0$ $\therefore$ $\frac{1}{3}t(3 - t^2) = 0$	M1	
		$t = 0$ (at O) or $\pm\sqrt{3}$; $t > 0$ $\therefore$ $a = \sqrt{3}$	A1	
	(b)	$\frac{dx}{dt} = 2t,\ \frac{dy}{dt} = 1 - t^2$	M1	
		$A = \int_0^{\sqrt{3}} 2\pi(t - \frac{1}{3}t^3)\sqrt{4t^2 + (1 - t^2)^2}\ dt$	M1 A1	
		$= 2\pi\int_0^{\sqrt{3}} (t - \frac{1}{3}t^3)\sqrt{1 + 2t^2 + t^4}\ dt$		
		$= 2\pi\int_0^{\sqrt{3}} (t - \frac{1}{3}t^3)(1 + t^2)\ dt$	M1	
		$= 2\pi\int_0^{\sqrt{3}} t + \frac{2}{3}t^3 - \frac{1}{3}t^5\ dt$	A1	
		$= 2\pi[\frac{1}{2}t^2 + \frac{1}{6}t^4 - \frac{1}{18}t^6]_0^{\sqrt{3}}$	A1	
		$= 2\pi[(\frac{3}{2} + \frac{3}{2} - \frac{3}{2}) - 0] = 3\pi$	M1 A1	**(10)**
5.	*(a)*	$2\cosh^2 x - 1 = 2 \times \frac{1}{4}(e^x + e^{-x})^2 - 1$	M1	
		$= \frac{1}{2}(e^{2x} + 2 + e^{-2x}) - 1$	M1	
		$= \frac{1}{2}(e^{2x} + e^{-2x}) + 1 - 1 = \frac{1}{2}(e^{2x} + e^{-2x}) = \cosh 2x$	A1	
	(b)	$2(2\cosh^2 x - 1) = 13\cosh x - 12$	M1	
		$4\cosh^2 x - 13\cosh x + 10 = 0$	A1	
		$(4\cosh x - 5)(\cosh x - 2) = 0$	M1	
		$\cosh x = \frac{5}{4}$ or 2	A1	
		$x = \ln(\frac{5}{4} + \sqrt{\frac{25}{16} - 1})$ or $\ln(2 + \sqrt{3})$	M1 A1	
		$x = \ln 2$ or $\ln(2 + \sqrt{3})$	A1	**(10)**
6.	*(a)*	$x^2 - 10x + 41 \equiv (x - 5)^2 - 25 + 41$	M1	
		$\equiv (x - 5)^2 + 16$ $\therefore$ $a = -5$, $b = 16$	A1	
	(b)	$\int_5^9 \frac{x}{\sqrt{x^2 - 10x + 41}}\ dx = \int_5^9 \frac{x}{\sqrt{(x - 5)^2 + 16}}\ dx$		
		$u = x - 5,\ \frac{du}{dx} = 1$	M1	
		$= \int_0^4 \frac{u + 5}{\sqrt{u^2 + 16}}\ du$	A1	
		$= \int_0^4 \frac{u}{\sqrt{u^2 + 16}} + \frac{5}{\sqrt{u^2 + 16}}\ du$	M1	
		$= [\sqrt{u^2 + 16} + 5\,\text{arsinh}(\frac{u}{4})]_0^4$	M1 A2	
		$= (\sqrt{32} + 5\,\text{arsinh}\ 1) - (4 + 0)$	M1	
		$= 4\sqrt{2} - 4 + 5\ln(1 + \sqrt{2})$		
		$= 4(\sqrt{2} - 1) + 5\ln(1 + \sqrt{2})$ $\therefore$ $p = 4$, $q = 5$, $r = 1 + \sqrt{2}$	A1	**(10)**

7.	*(a)*	$u = x^n,\ u' = nx^{n-1};\quad v' = \cos x,\ v = \sin x$	M1	
		$I_n = [x^n \sin x]_0^{\frac{\pi}{2}} - n\int_0^{\frac{\pi}{2}} x^{n-1}\sin x\ \mathrm{d}x$	A1	
		$u = x^{n-1},\ u' = (n-1)x^{n-2};\quad v' = \sin x,\ v = -\cos x$	M1	
		$I_n = [x^n \sin x]_0^{\frac{\pi}{2}} - n\{\ [-x^{n-1}\cos x]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} (n-1)x^{n-2}\cos x\ \mathrm{d}x\ \}$	A1	
		$I_n = (\frac{\pi}{2})^n - 0 - n(0-0) - n(n-1)I_{n-2}$		
		$I_n = (\frac{\pi}{2})^n - n(n-1)I_{n-2}$	A1	
	(b)	$I_0 = \int_0^{\frac{\pi}{2}} \cos x\ \mathrm{d}x\ =\ [\sin x]_0^{\frac{\pi}{2}} = 1$	M1 A1	
		$I_2 = (\frac{\pi}{2})^2 - 2\times 1\times I_0 = \frac{1}{4}\pi^2 - 2$	M1 A1	
		$I_4 = (\frac{\pi}{2})^4 - 4\times 3\times(\frac{1}{4}\pi^2 - 2) = \frac{1}{16}\pi^4 - 3\pi^2 + 24$	M1 A1	**(11)**
8.	*(a)*	$y = \frac{c^2}{x},\ \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{c^2}{x^2}$	M1	
		at P, $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{c^2}{c^2p^2} = -\frac{1}{p^2}$	A1	
		eqn. is $y - \frac{c}{p} = -\frac{1}{p^2}(x - cp)$	M1	
		giving $x + yp^2 = 2cp$	A1	
	(b)	X is $(cq, 0)$	B1	
		$\therefore\ cq + 0 = 2cp$ giving $q = 2p$	M1 A1	
	(c)	$P(cp, \frac{c}{p}),\ Q(2cp, \frac{c}{2p})$	M1	
		x-coord of $M = \frac{1}{2}(cp + 2cp) = \frac{3}{2}cp$	A1	
		y-coord of $M = \frac{1}{2}(\frac{c}{p} + \frac{c}{2p}) = \frac{3c}{4p}$	A1	
		$x\times y = \frac{3}{2}cp\times\frac{3c}{4p}\quad\therefore$ eqn is $xy = \frac{9c^2}{8}$	M1 A1	**(12)**
			Total	**(75)**

Performance Record – FP2 Paper A

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	rad. of curv.	integr. hyp. fns	diff. inv. trig.	surface area	eqn. in hyp. fns.	integr. std. forms	reduction formula	hyperbola, tangent, loci	
Marks	5	8	9	10	10	10	11	12	75
Student									