edexcel

June 2009
6675 Further Pure Mathematics FP2 (legacy)
Mark Scheme

Question Number	Scheme	Marks
Q1	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2\times \operatorname{arsinh} 2x \times \frac{2}{\sqrt{(4x^2+1)}}$	M1 A1
	At $x=\frac{1}{2}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{\sqrt{2}}\operatorname{arsinh} 1$	M1 A1ft
	$= 2\sqrt{2}\ln\left(\sqrt{2}+1\right)$	A1 (5)
		[5]
	Alternative	
	$\sinh y^{\frac{1}{2}} = 2x$	
	$\frac{1}{2}y^{-\frac{1}{2}}\cosh y^{\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}x} = 2$	M1 A1
	$\sqrt{\left(1+\sinh^2 y^{\frac{1}{2}}\right)}\frac{\mathrm{d}y}{\mathrm{d}x} = 4y^{\frac{1}{2}}$	
	At $x=\frac{1}{2}$, $\sinh y^{\frac{1}{2}} = 1$	
	$\sqrt{(1+1)}\frac{\mathrm{d}y}{\mathrm{d}x} = 4\operatorname{arsinh} 1$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{\sqrt{2}}\operatorname{arsinh} 1$	A1ft
	$= 2\sqrt{2}\ln\left(\sqrt{2}+1\right)$	A1 (5)
Q2 (a)	$b^2 = a^2(1-e^2) \Rightarrow 8 = a^2\left(1-\frac{1}{2}\right) \Rightarrow a = 4$	M1 A1 (2)
(b)	At S, $x = ae = 2\sqrt{2}$; at D, $y = 2\sqrt{2}$ two coordinates	B1
	($SDS'D'$ is a square)	
	$A = 4\times\frac{1}{2}\times 2\sqrt{2}\times 2\sqrt{2} = 16$	M1 A1 (3)
		[5]

Question Number	Scheme	Marks
Q3 (a)	$\int_0^1 (1-x)^n \cosh x \, dx = \left[(1-x)^n \sinh x\right]_0^1 + \int_0^1 n(1-x)^{n-1} \sinh x \, dx$	M1
	$= \int_0^1 n(1-x)^{n-1} \sinh x \, dx$	A1
	$= \left[n(1-x)^{n-1} \cosh x\right]_0^1 + \int_0^1 n(n-1)(1-x)^{n-2} \cosh x \, dx$	M1
	$= -n + n(n-1)\int_0^1 (1-x)^{n-2} \cosh x \, dx$	M1
	$I_n = n(n-1)I_{n-2} - n$ ✱ cso	A1 (5)
(b)	$I_0 = \int_0^1 \cosh x \, dx = \left[\sinh x\right]_0^1 = \sinh 1 \quad \left(= \tfrac{1}{2}\left(e - e^{-1}\right)\right)$	B1
	$I_2 = 2I_0 - 2$	M1
	$I_4 = 12I_2 - 4 = 24I_0 - 28$	M1
	$= 12e - \dfrac{12}{e} - 28$	A1 (4)
		[9]
Q4 (a)	$\dfrac{dy}{dx} = 15\cosh x - 17\sinh x + 6$	B1
	$\dfrac{dy}{dx} = 0 \Rightarrow 15\left(\dfrac{e^x + e^{-x}}{2}\right) - 17\left(\dfrac{e^x - e^{-x}}{2}\right) + 6 = 0$	M1
	$e^{2x} - 6e^x - 16 = 0$	M1 A1
	$\left(e^x - 8\right)\left(e^x + 2\right) = 0$	M1
	$x = 3\ln 2$	A1 (6)
(b)	$\dfrac{d^2y}{dx^2} = 15\sinh x - 17\cosh x$	M1
	$= -e^x - 16e^{-x} < 0$ (for any real x)	M1
	$\Rightarrow$ maximum	A1 (3)
	Accept equivalent arguments or a sketch	
		[9]

Question Number	Scheme		Marks	
Q5	Use of $S = 2\pi\int y\left(\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2\right)^{\frac{1}{2}}\mathrm{d}t$		B1	
	$\int y\sqrt{\left(\dot{x}^2+\dot{y}^2\right)}\mathrm{d}t = \int 3t^2\sqrt{\left(36t^4+36t^2\right)}\mathrm{d}t$		M1 A1	
	$= \int 18t^3\sqrt{\left(t^2+1\right)}\mathrm{d}t$			
	Let $u^2 = t^2+1$, $u\frac{\mathrm{d}u}{\mathrm{d}t} = t$			
	$\int t^3\sqrt{\left(t^2+1\right)}\mathrm{d}t = \int\left(u^2-1\right)u^2\,\mathrm{d}u$		M1 A1	
	$= \left(\frac{u^5}{5}-\frac{u^3}{3}\right)$		M1 A1	
	$\left[\left(\frac{u^5}{5}-\frac{u^3}{3}\right)\right]_1^{\sqrt{2}} = \frac{1}{15}\left(2\sqrt{2}-(-2)\right)$	using correct limits	M1	
	Leading to $A = \frac{24\pi}{5}\left(\sqrt{2}+1\right)$ ✱	cso	A1	(9)
				[9]
	Alternative substitutions			
	① Let $u = t^2+1$, $\frac{\mathrm{d}u}{\mathrm{d}t} = 2t$			
	$\int t^3\sqrt{\left(t^2+1\right)}\mathrm{d}t = \frac{1}{2}\int(u-1)u^{\frac{1}{2}}\,\mathrm{d}u$		M1 A1	
	$= \frac{1}{2}\int\left(u^{\frac{3}{2}}-u^{\frac{1}{2}}\right)\mathrm{d}u = \frac{1}{2}\left(\frac{2}{5}u^{\frac{5}{2}}-\frac{2}{3}u^{\frac{3}{2}}\right)$		M1 A1	
	Using the limits $u=1$ and $u=2$		M1	
	Leading to $A = \frac{24\pi}{5}\left(\sqrt{2}+1\right)$ ✱	cso	A1	
	② Let $t = \sinh u$, $\frac{\mathrm{d}t}{\mathrm{d}u} = \sinh u$			
	$\int t^3\sqrt{\left(t^2+1\right)}\mathrm{d}t = \int \sinh^3 u\cosh^2 u\,\mathrm{d}u$		M1 A1	
	$= \int\left(\cosh^4 u-\cosh^2 u\right)\sinh u\,\mathrm{d}u = \frac{\cosh^5 u}{5}-\frac{\cosh^3 u}{3}$		M1 A1	
	Using the limits $\cosh u = 1$ and $\cosh u = \sqrt{2}$		M1	
	Leading to $A = \frac{24\pi}{5}\left(\sqrt{2}+1\right)$ ✱	cso	A1	

Question Number	Scheme	Marks
Q6	$u = \cosh\theta \quad \Rightarrow \quad \dfrac{du}{d\theta} = \sinh\theta$	B1
	$I = \displaystyle\int \frac{u+1}{\sinh^2\theta(u-1)^2}\,du$	M1
	$= \displaystyle\int \frac{u+1}{(u^2-1)(u-1)^2}\,du$	M1
	$= \displaystyle\int \frac{1}{(u-1)^3}\,du$	A1
	$= -\dfrac{1}{2(u-1)^2}$	M1 A1
	At $\theta = \ln 4$, $u = \dfrac{4+\frac{1}{4}}{2} = \dfrac{17}{8}$; at $\theta = \ln 2$, $u = \dfrac{2+\frac{1}{2}}{2} = \dfrac{5}{4}$ both	M1 A1
	$\left[-\dfrac{1}{2(u-1)^2}\right]_{5/4}^{17/8} = 8 - \dfrac{32}{81} = \dfrac{616}{81}$	M1 A1 (10)
		[10]

Question Number	Scheme	Marks	
Q7 (a)	$\frac{dy}{dx} = \frac{\cos x}{\sin x} \left(= \cot x\right)$	B1	
	$\frac{dy}{dx} = \tan\psi = \cot x$	M1	
	$\tan\psi = \tan\left(\frac{\pi}{2} - x\right) \Rightarrow \psi = \frac{\pi}{2} - x$ ✱ cso	A1	(3)
(b)	$s = \int \left(1 + \left(\frac{dy}{dx}\right)^2\right)^{1/2} dx = \int \left(1 + \cot^2 x\right)^{1/2} dx$	M1	
	$= \int \operatorname{cosec} x \, dx$	A1	
	$= -\ln\left(\operatorname{cosec} x + \cot x\right) (+C)$	A1	
	$= -\ln\left(\sec\psi + \tan\psi\right) (+C)$	M1	
	$\left(0, \frac{\pi}{4}\right) \Rightarrow 0 = -\ln\left(\sqrt{2} + 1\right) + C$	M1	
	$s = \ln\left(\frac{\sqrt{2} + 1}{\sec\psi + \tan\psi}\right)$ ✱ cso	A1	(6)
(c)	$\frac{ds}{d\psi} = -\sec\psi$	M1	
	$\psi = \frac{\pi}{6} \Rightarrow \rho = \left\lvert \frac{ds}{d\psi} \right\rvert = \frac{2}{\sqrt{3}}$ awrt 1.15	M1 A1	(3)
			[12]
	Alternative to (c)		
	$\psi = \frac{\pi}{6} \Rightarrow x = \frac{\pi}{3}$		
	At $x = \frac{\pi}{3}$; $\frac{dy}{dx} = \cot x = \frac{1}{\sqrt{3}}$, $\frac{d^2 y}{dx^2} = -\operatorname{cosec}^2 x = -\frac{4}{3}$ both	M1	
	$\rho = \left\lvert \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2 y}{dx^2}} \right\rvert = \frac{\left(1 + \frac{1}{3}\right)^{\frac{3}{2}}}{\frac{4}{3}} = \frac{2}{\sqrt{3}}$ awrt 1.15	M1 A1	(3)

Question Number	Scheme		Marks	
Q8 (a)	$\frac{dx}{dp} = 2ap,\ \frac{dy}{dp} = 2a;\quad \frac{dy}{dx} = \frac{1}{p}$		M1 A1	
	$y - 2ap = -p\left(x - ap^2\right)$		M1	
	$y + px = 2ap + ap^3$ ✱	cso	A1	(4)
(b)	Eliminating x between $y^2 = 4ax$ and $y + px = 2ap + ap^3$			
	$y + \frac{py^2}{4a} = 2ap + ap^3$		M1	
	$py^2 + 4ay - 8a^2p - 4a^2p^3 = 0$		A1	
	$\left(y - 2ap\right)\left(py + 4a + 2ap^2\right) = 0$		M1 A1	
	At Q, $y = -\frac{4a + 2ap^2}{p} = -2a\left(\frac{2 + p^2}{p}\right)$ ✱	cso	A1	(5)
(c)	At Q, $x = a\left(\frac{2 + p^2}{p}\right)^2$			
	$PQ^2 = \left(ap^2 - a\left(\frac{2 + p^2}{p}\right)^2\right)^2 + \left(2ap + 2a\left(\frac{2 + p^2}{p}\right)\right)^2$		M1 A1	
	$= \frac{16a^2\left(p^2 + 1\right)^3}{p^4}$			
	$\frac{d}{dx}\left(PQ^2\right) = 16a^2\left(\frac{6\left(p^2 + 1\right)^2 p^5 - \left(p^2 + 1\right)^3 .4p^3}{p^8}\right)$		M1	
	$\frac{d}{dx}\left(PQ^2\right) = 0 \quad \Rightarrow \quad \frac{2\left(p^2 + 1\right)^2\left(p^2 - 2\right)}{p^5} = 0$			
	$p = \left(\pm\right)\sqrt{2}$		M1 A1	
	$PQ^2 = \frac{16a^2 \times 27}{4}$		M1	
	$PQ_{\min} = 6\sqrt{3}a$ ✱	cso	A1	(7)
				[16]