Mark Scheme (Results)

Summer 2007

GCE

GCE Mathematics

Further Pure Mathematics FP1 (6674)

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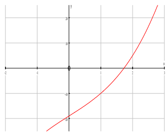
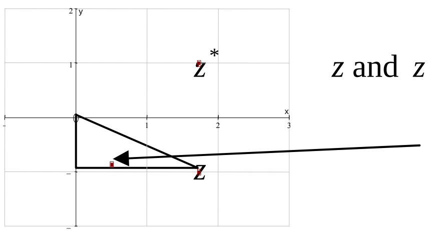
Question number	Scheme	Marks
1.	$1\frac{1}{2}$ and 3 are 'critical values', e.g. used in solution, or both seen as asymptotes	B1
	$(x+1)(x-3) = 2x-3 \Rightarrow \quad x(x-4) = 0$	
	$x = 4, \quad x = 0$ M1: attempt to find at least one other critical value	M1 A1, A1
	$0 < x < 1\frac{1}{2}, \quad 3 < x < 4$ M1: An inequality using $1\frac{1}{2}$ or 3	M1 A1, A1 (7)
		7
	First M mark can be implied by the two correct values, but otherwise a method must be seen. (The method may be graphical, but either (x =) 4 or (x =) 0 needs to be clearly written or used in this case). Ignore 'extra values' which might arise through 'squaring both sides' methods. ≤ appearing: maximum one A mark penalty (final mark).	

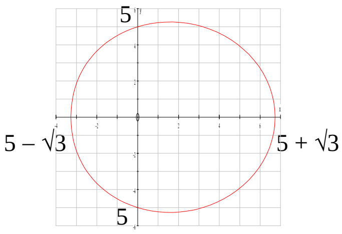
Question number	Scheme	Marks
2.	Integrating factor $e^{\int -\tan x \, dx} = e^{\ln(\cos x)} \left(\text{or } e^{-\ln(\sec x)}\right)$, $= \cos x \left(\text{or } \frac{1}{\sec x}\right)$	M1, A1
	$\left(\cos x \frac{dy}{dx} - y \sin x = 2\sec^2 x\right)$	
	$y\cos x = \int 2\sec^2 x \, dx$ (or equiv.) $\left(\text{Or}: \frac{d}{dx}(y\cos x) = 2\sec^2 x\right)$	M1 A1(ft)
	$y\cos x = 2\tan x \ (+C)$ (or equiv.)	A1
	$y = 3$ at $x = 0$: $C = 3$	M1
	$y = \frac{2\tan x + 3}{\cos x}$ (Or equiv. in the form $y = f(x)$)	A1 (7)
		7
	1st M: Also scored for $e^{\int \tan x \, dx} = e^{-\ln(\cos x)} \left(\text{or } e^{\ln(\sec x)}\right)$, then A0 for $\sec x$. 2nd M: Attempt to use their integrating factor (requires one side of the equation 'correct' for their integrating factor). 2nd A: The follow-through is allowed <u>only</u> in the case where the integrating factor used is sec x or $-\sec x$. $\left(y\sec x = \int 2\sec^4 x \, dx\right)$ 3rd M: Using $y = 3$ at $x = 0$ to find a value for C (dependent on an integration attempt, however poor, on the RHS). <u>Alternative</u> 1st M: Multiply through the given equation by $\cos x$. 1st A: Achieving $\cos x \frac{dy}{dx} - y\sin x = 2\sec^2 x$. (Allowing the possibility of integrating by inspection).	

Question number	Scheme	Marks
3.	(a) $(r+1)^3 = r^3 + 3r^2 + 3r + 1$ and $(r-1)^3 = r^3 - 3r^2 + 3r - 1$	M1
	$(r+1)^3 - (r-1)^3 = 6r^2 + 2$ (*)	A1cso (2)
	(b) $r = 1: \quad 2^3 - 0^3 = 6(1^2) + 2$	
	$r = 2: \quad 3^3 - 1^3 = 6(2^2) + 2$	
	: : : : : : : : : : :	
	$r = n: \quad (n+1)^3 - (n-1)^3 = 6n^2 + 2$ M: Differences: at least first, last and one other.	M1 A1
	Sum: $(n+1)^3 + n^3 - 1 = 6\sum r^2 + 2n$ M: Attempt to sum at least one side.	M1 A1
	$\left(6\sum r^2 = 2n^3 + 3n^2 + n\right)$	
	$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ (Intermediate steps are not required) (*)	A1cso (5)
	(c) $\sum_{r=n}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n-1} r^2, \ = \frac{1}{6}(2n)(2n+1)(4n+1) - \frac{1}{6}(n-1)n(2n-1)$	M1, A1
	$= \frac{1}{6}n\left(\left(16n^2 + 12n + 2\right) - \left(2n^2 - 3n + 1\right)\right)$	M1
	$= \frac{1}{6}n(n+1)(14n+1)$	A1 (4)
		11
	(b) 1st A: Requires first, last and one other term correct on both LHS and RHS (but condone 'omissions' if following work is convincing). (c) 1st M: Allow also for $\sum_{r=n}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n} r^2$. 2nd M: Taking out (at some stage) factor $\frac{1}{6}n$, and multiplying out brackets to reach an expression involving n^2 terms.	

Question number	Scheme	Marks	
4.	(a) $f'(x) = 3x^2 + 8$ $\quad 3x^2 + 8 = 0 \ldots\ldots$ or $3x^2 + 8 > 0 \ldots\ldots$ Correct derivative and, e.g., 'no turning points' or 'increasing function'.	M1 A1	
	 Simple sketch, (increasing, crossing positive x-axis) (or, if the M1 A1 has been scored, a <u>reason</u> such as 'crosses x-axis only once').	B1	(3)
	(b) Calculate $f(1)$ and $f(2)$ (<u>Values</u> must be seen) $f(1) = -10$, $f(2) = 5$, Sign change, $\therefore$ Root	M1 A1	 (2)
	(c) $x_1 = 2 - \frac{f(2)}{f'(2)}$, $\quad = 2 - \frac{5}{20} \quad (= 1.75)$	M1, A1	
	$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$, $\quad \left(= 1.75 - \frac{0.359375}{17.1875}\right) \quad = 1.729$ (ONLY) (α)	M1, A1	(4)
	(d) Calculate $f(\alpha - 0.0005)$ and $f(\alpha + 0.0005)$ (or a 'tighter' interval that gives a sign change). $f(1.7285) = -0.0077\ldots$ and $f(1.7295) = 0.0092\ldots$, $\therefore$ Accurate to 3 d.p.	M1 A1	 (2)
			11
	(a) M: Differentiate and consider sign of $f'(x)$, or equate $f'(x)$ to zero. <u>Alternative</u>: M1: Attempt to rearrange as $x^3 - 19 = -8x$ or $x^3 = 19 - 8x$ (condone sign slips), and to sketch a cubic graph and a straight line graph. A1: Correct graphs (shape correct and intercepts 'in the right place'). B1: Comment such as "one intersection, therefore one root"). (c) 1st A1 can be implied by an answer of 1.729, provided N.R. has been used. <u>Answer only</u>: No marks. The Newton-Raphson method must be seen. (d) For A1, correct <u>values</u> of f(1.7285) and f(1.7295) must be seen, together with a conclusion. If only 1 s.f. is given in the values, allow rounded (e.g. -0.008) <u>or</u> truncated (e.g. -0.007) values.		

Question number	Scheme	Marks	
5.	C.F. $m^2+3m+2=0$ $\quad m=-1$ and $m=-2$	M1	
	$y=Ae^{-x}+Be^{-2x}$	A1	(2)
	P.I. $y=cx^2+dx+e$	B1	
	$\frac{dy}{dx}=2cx+d$, $\frac{d^2y}{dx^2}=2c$ $\quad 2c+3(2cx+d)+2(cx^2+dx+e)\equiv 2x^2+6x$	M1	
	$2c=2$ $\quad c=1$ $\quad$ (One correct value)	A1	
	$6c+2d=6$ $\quad d=0$		
	$2c+3d+2e=0$ $\quad e=-1$ $\quad$ (Other two correct values)	A1	
	General soln: $y=Ae^{-x}+Be^{-2x}+x^2-1$ $\quad$ (Their C.F. + their P.I.)	A1ft	(5)
	$x=0,\ y=1:\ \ 1=A+B-1$ $\quad (A+B=2)$	M1	
	$\frac{dy}{dx}=-Ae^{-x}-2Be^{-2x}+2x$, $\quad x=0,\ \frac{dy}{dx}=1:$ $\quad 1=-A-2B$	M1	
	Solving simultaneously: $A=5$ and $B=-3$	M1 A1	
	Solution: $y=5e^{-x}-3e^{-2x}+x^2-1$	A1	(5)
			12
	1st M: Attempt to solve auxiliary equation. 2nd M: Substitute their $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ into the D.E. to form an identity in x with unknown constants. . 3rd M: Using y = 1 at x = 0 in their general solution to find an equation in A and B. 4th M: Differentiating their general solution (condone 'slips', but the powers of each term must be correct) and using $\frac{dy}{dx}=1$ at x = 0 to find an equation in A and B. 5th M: Solving simultaneous equations to find both a value of A and a value of B.		

Question number	Scheme	Marks
6.	(a) $z^* = \sqrt{3} + \mathrm{i}$	B1
	$\frac{z}{z^*} = \frac{(\sqrt{3}-\mathrm{i})(\sqrt{3}-\mathrm{i})}{(\sqrt{3}+\mathrm{i})(\sqrt{3}-\mathrm{i})} = \frac{3-2\sqrt{3}\mathrm{i}-1}{3+1}, \quad = \frac{1}{2} - \frac{\sqrt{3}}{2}\mathrm{i}$ (*)	M1, A1cso (3)
	(b) $\left\lvert \frac{z}{z^*} \right\rvert = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\pm\sqrt{3}}{2}\right)^2}, \quad = 1$ $\quad \left[\text{Or: } \left\lvert \frac{z}{z^*} \right\rvert = \frac{\lvert z \rvert}{\lvert z^* \rvert} = \frac{\sqrt{3+1}}{\sqrt{3+1}}, \quad = 1 \right]$	M1, A1 (2)
	(c) $\arg(w) = \arctan\left(\pm \frac{\text{imag}(w)}{\text{real}(w)}\right)$ or $\arg(w) = \arctan\left(\pm \frac{\text{real}(w)}{\text{imag}(w)}\right)$, where w is z or z^* or $\frac{z}{z^*}$	M1
	$\arg\left(\frac{z}{z^*}\right) = \arctan\left(\frac{-\sqrt{3}/2}{1/2}\right) \quad = -\frac{\pi}{3}$	A1
	$\arctan\left(\frac{-1}{\sqrt{3}}\right) = -\frac{\pi}{6}$ and $\arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$ (Ignore interchanged z and z^*)	A1
	$\arg z - \arg z^* = -\frac{\pi}{6} - \frac{\pi}{6} = -\frac{\pi}{3} = \arg\left(\frac{z}{z^*}\right)$	A1 (4)
	(d)  z and z^* (Correct quadrants, approx. symmetrical)	B1
	$\frac{z}{z^*}$ (Strictly <u>inside</u> the triangle shown here)	B1 (2)
	(e) $(x - (\sqrt{3} - \mathrm{i}))(x - (\sqrt{3} + \mathrm{i}))$	M1
	Or: Use sum of roots $\left(= \frac{-b}{a}\right)$ and product of roots $\left(= \frac{c}{a}\right)$.	
	$x^2 - 2\sqrt{3}x + 4$	A1 (2) **13**
	(a) M: Multiplying both numerator and denominator by $\sqrt{3} - \mathrm{i}$, and multiplying out brackets with <u>some</u> use of $\mathrm{i}^2 = -1$. (b) Answer 1 with no working scores both marks. (c) Allow work in degrees: –60°, –30° and 30° Allow arg between 0 and 2π: $\frac{5\pi}{3}$, $\frac{11\pi}{6}$ and $\frac{\pi}{6}$ (or 300°, 330° and 30°). Decimals: Allow marks for awrt –1.05 (A1), –0.524 and 0.524 (A1), but then A0 for final mark. (Similarly for 5.24 (A1), 5.76 and 0.524 (A1)). (d) Condone wrong labelling (or lack of labelling), if the intention is clear.	

Question number	Scheme	Marks
7.	(a)  Shape (closed curve, approx. symmetrical about the initial line, in all ‘quadrants’ and ‘centred’ to the right of the pole/origin).	B1
	Scale (at least one correct ‘intercept’ r value… shown on sketch or perhaps seen in a table). (Also allow awrt 3.27 or awrt 6.73).	B1 (2)
	(b) $y = r\sin\theta = 5\sin\theta + \sqrt{3}\sin\theta\cos\theta$	M1
	$\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = 5\cos\theta - \sqrt{3}\sin^2\theta + \sqrt{3}\cos^2\theta \quad \left(= 5\cos\theta + \sqrt{3}\cos 2\theta\right)$	A1
	$5\cos\theta - \sqrt{3}\left(1-\cos^2\theta\right) + \sqrt{3}\cos^2\theta = 0$	M1
	$2\sqrt{3}\cos^2\theta + 5\cos\theta - \sqrt{3} = 0$	
	$\left(2\sqrt{3}\cos\theta - 1\right)\left(\cos\theta + \sqrt{3}\right) = 0 \qquad \cos\theta = \ldots \quad (0.288\ldots)$	M1
	$\theta = 1.28$ and 5.01 (awrt) (Allow ±1.28 awrt) $\left(\text{Also allow} \pm \arccos\dfrac{1}{2\sqrt{3}}\right)$	A1
	$r = 5 + \sqrt{3}\left(\dfrac{1}{2\sqrt{3}}\right) = \dfrac{11}{2}$ (Allow awrt 5.50)	A1 (6)
	(c) $r^2 = 25 + 10\sqrt{3}\cos\theta + 3\cos^2\theta$	B1
	$\int 25 + 10\sqrt{3}\cos\theta + 3\cos^2\theta \, \mathrm{d}\theta = \underline{\dfrac{53\theta}{2}} + 10\sqrt{3}\sin\theta + \underline{\underline{3\left(\dfrac{\sin 2\theta}{4}\right)}}$	M1 <u>A1ft</u> A1ft (double underlined)
	(ft for integration of $(a + b\cos\theta)$ and $c\cos 2\theta$ respectively)	
	$\dfrac{1}{2}\left[25\theta + 10\sqrt{3}\sin\theta + \dfrac{3\sin 2\theta}{4} + \dfrac{3\theta}{2}\right]_0^{2\pi} = \ldots\ldots$	M1
	$= \dfrac{1}{2}(50\pi + 3\pi) = \dfrac{53\pi}{2}$ or equiv. in terms of π.	A1 (6)
		14
	(b) 2nd M: Forming a quadratic in $\cos\theta$. 3rd M: Solving a 3 term quadratic to find a value of $\cos\theta$ (even if called θ). <u>Special case</u>: Working with $r\cos\theta$ instead of $r\sin\theta$: 1st M1 for $r\cos\theta = 5\cos\theta + \sqrt{3}\cos^2\theta$ 1st A1 for derivative $-5\sin\theta - 2\sqrt{3}\sin\theta\cos\theta$, then no further marks. (c) 1st M: Attempt to integrate at least one term. 2nd M: Requires use of the $\dfrac{1}{2}$, correct limits (which could be 0 to 2π, or $-\pi$ to π, or ‘double’ 0 to π), and subtraction (which could be implied).	