Mark Scheme (Final)

Summer 2009

GCE

GCE Decision Mathematics D2 (6690/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

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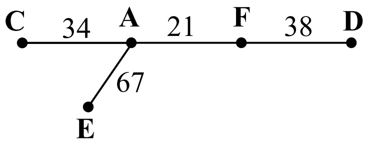
Mark Scheme

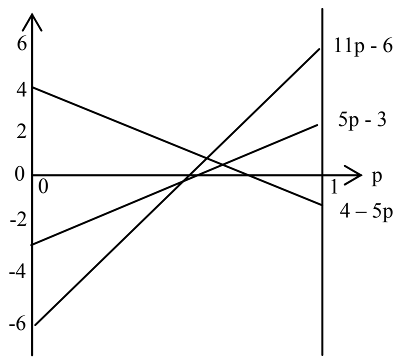
Question Number	Scheme	Marks
Q1		
(a)	There are more tasks than people.	B1 (1)
(b)	Adds a row of zeros	B1 (1)
(c)	$\begin{bmatrix} 15 & 11 & 14 & 12 \\ 13 & 8 & 17 & 13 \\ 14 & 9 & 13 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 0 & 3 & 1 \\ 5 & 0 & 9 & 5 \\ 5 & 0 & 4 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \rightarrow \begin{bmatrix} 3 & 0 & 2 & 0 \\ 4 & 0 & 8 & 4 \\ 4 & 0 & 3 & 5 \\ 0 & 1 & 0 & 0 \end{bmatrix}$	B1;M1A1
	Either $\begin{bmatrix} 3 & 3 & 2 & 0 \\ 1 & 0 & 5 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 4 & 0 & 0 \end{bmatrix}$	M1 A1
	Or $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 0 & 6 & 4 \\ 2 & 0 & 1 & 5 \\ 0 & 3 & 0 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 5 & 3 \\ 1 & 0 & 0 & 4 \\ 0 & 4 & 0 & 2 \end{bmatrix}$	
	J – 4, M – 2, R – 3, (D – 1)	A1 (6)
(d)	Minimum cost is (£)33.	B1 (1)
		Total 9

Question Number	Scheme	Marks
Q2		
(a)	In the classical problem each vertex must be visited **only** once. In the practical problem each vertex must be visited **at least** once.	 B2, 1, 0 (2)
(b)	A F D B E C A {1 4 6 3 5 2 } 21 + 38 + 58 + 36 + 70 + 34 = 257	M1 A1 A1 (3)
(c)	257 is the better upper bound, it is lower.	B1ft (1)
(d)	R.M.S.T.  Lower bound is 160 + 36 + 58 = 254	M1 A1 M1A1 (4)
(e)	Better lower bound is 254, it is higher	B1ft
(f)	$254 < \text{optimal} \le 257$	B1 (2)
	Notes: (a) 1B1: Generous, on the right lines bod gets B1 2B1: cao, clear answer. (b) 1M1:Nearest Neighbour each vertex visited once (condone lack of return to start) 1A1: Correct route cao – must return to start. 2A1: 257 cao (c) 1B1ft: ft their lowest. (d) 1M1: Finding correct RMST (maybe implicit) 160 sufficient 1A1: cao tree or 160. 2M1:Adding 2 least arcs to B, 36 and 58 only 2A1: 254 (e) 1B1ft: ft their highest (f) 1B1: cao	**Total 12**

Question Number	Scheme	Marks
Q3		
(a)	Row minima {-5, -4, -2} row maximin = -2 Column maxima {1, 6, 13} col minimax = 1 $-2 \neq 1$ therefore not stable.	M1 A1 A1 (3)
(b)	Column 1 dominates column 3, so column 3 can be deleted.	B1 (1)
(c)	see table below	B1 B1 (2)
(d)	Let B play row 1 with probability p and row 2 with probability (1-p) If A plays 1, B's expected winnings are 11p – 6 If A plays 2, B's expected winnings are 4 – 5p If A plays 3, B's expected winnings are 5p – 3	M1 A1
	see graph below	M1 A1
	$5p-3=4-5p$ $10p=7$ $p=\frac{7}{10}$	M1
	B should play 1 with a probability of 0.7 2 with a probability of 0.3 and never play 3	A1
	The value of the game is 0.5 to B	A1 (7)
		Total 13

(c)

	A plays 1	A plays 2	A plays 3
B plays 1	5	-1	2
B plays 2	-6	4	-3



Question Number	Scheme	Marks
Q4		
(a)	Value of cut $C_1 = 34$; Value of cut $C_2 = 45$	B1; B1 (2)
(b)	S B F G T or S B F E T – value 2 Maximum flow = 28	M1 A1 A1=B1 (3)
	Notes: (a) 1B1: cao 2B1: cao (b) 1M1: feasible flow-augmenting route and a value stated 1A1: a correct flow-augmenting route and value 1A1= B1: cao	**Total 5**

Question Number	Scheme	Marks
Q5		
(a)	$x = 0,\ y = 0,\ z = 2$	B2,1,0 (2)
(b)	$P - 2x - 4y + \frac{5}{4}r = 10$	M1 A1 (2)
		Total 4
	Notes: (a) 1B1: Any 2 out of 3 values correct 2B1: All 3 values correct. (b) 1M1: One equal sign, modulus of coefficients correct. All the right ingredients. 1A1: cao – condone terms of zero coefficient	

Question Number	Scheme	Marks
Q6 (a)	The supply is equal to the demand	B1 (1)
(b)	see table below	B1 (1)

	A	B	C
X	16	6	
Y		9	8
Z			15

(c)

	A	B	C
X	16- θ	6+ θ	
Y		9- θ	8+ θ
Z	θ		15- θ

Value of θ = 9, exiting cell is YB

M1 A1

A1 (3)

(d)

		17	8	20
		A	B	C
0	X	7	15	
-5	Y			17
-11	Z	9		6

M1 A1

XC = 7 – 0 – 20 = -13
YA = 16 + 5 – 17 = 4
YB = 12 + 5 – 8 = 9
ZB = 10 + 11 – 8 = 13

A1 (3)

	A	B	C
X	7- θ	15	θ
Y			17
Z	9+ θ		6- θ

Value of θ = 6, entering cell XC, exiting cell ZC

M1 A1

	A	B	C
X	1	15	6
Y			17
Z	15		

A1 (3)

Cost (£) 524

B1 (1)

Total 12

Question Number	Scheme	Marks
Q7 (a)		

Stage	State (in £1000s)	Action (in £1000s)	Dest. (in £1000s)	Value (in £1000s)	Marks
	250	*250*	*0*	*300**	
1	*200*	*200*	*0*	*240**	
	150	*150*	*0*	*180**	
	100	*100*	*0*	*120**	
	50	*50*	*0*	*60**	
	0	*0*	*0*	*0**	
	250	*280*	*0*	*200 + 0 = 280*	
		200	*50*	*235 + 60 = 295*	
		150	*100*	190 + 120 = 310*	
		100	*150*	125 + 180 = 305	1M1 A1
		50	*200*	65 + 240 = 305	
		0	*250*	0 + 300 = 300	
2	*200*	*200*	*0*	235 + 0 = 235	
		150	*50*	190 + 60 = 250*	
		100	100	125 + 120 = 245	A1
		50	150	65 + 180 = 245	
		0	200	0 + 240 = 240	
	150	*150*	0	190 + 0 = 190*	2M1
		100	50	125 + 60 = 185	
		50	100	65 + 120 = 185	A1
		0	150	0 + 180 = 180	
	100	100	0	125 + 0 = 125*	A1
		50	50	65 + 60 = 125*	
		0	100	0 + 120 = 120	
	50	50	0	65 + 0 = 65*	
		0	50	0 + 60 = 60	
	0	0	0	0 + 0 = 0*	3M1
3	250	250	0	300 + 0 = 300	A1ft
		200	50	230 + 65 = 295	
		150	100	170 + 125 = 295	
		100	150	110 + 190 = 300	
		50	200	55 + 250 = 305	
		0	250	0 + 310 = 310*	

Maximum income £310 000 — B1

Scheme	1	2	3
Invest (in £1000s)	100	150	0

B1 (10)

(b)

Stage: Scheme being considered — B1

State: Money available to invest — B1

Action: Amount chosen to invest — B1 (3)

Total 13

Question Number	Scheme	Marks
Q8	E.g. Add 6 to make all elements positive $\begin{bmatrix} 4 & 14 & 5 \\ 13 & 10 & 3 \\ 7 & 1 & 10 \end{bmatrix}$	B1
	Let Laura play 1, 2 and 3 with probabilities p_1, p_2 and p_3 respectively Let V = value of game + 6	B1
	e.g. Maximise P = V Subject to: $V-4p_1-13p_2-7p_3\le 0$ $V-14p_1-10p_2-p_3\le 0$ $V-5p_1-3p_2-10p_3\le 0$ $p_1+p_2+p_3\le 1$ $p_1,p_2,p_3\ge 0$	B1 M1 A3,2ft,1ft,0 (7)
	Notes: 1B1: Making all elements positive 2B1: Defining variables 3B1: Objective, cao word and function 1M1: At least one constraint in terms of their variables, must be going down columns. Accept = here. 1A1ft: ft their table. One constraint in V correct. 2A1ft: ft their table. Two constraints in V correct. 3A1: CAO all correct .	**Total 7**
	Alt using x_i method Now additionally need: let $x_i=\frac{p_i}{v}$ for 2B1 minimise $(P)=x_1+x_2+x_3=\frac{1}{v}$ subject to: $4x_1+13x_2+7x_3\ge 1$ $14x_1+10x_2+x_3\ge 1$ $5x_1+3x_2+10x_3\ge 1$ $x_i\ge 0$	

June 2009 Decision Mathematics 2 6690/01

Question 1 notes:

(a) 1B1: cao
(b) 1B1: cao
(c) 1B1: row then column reductions correct
1M1: Double covered +e; one uncovered – e; and one single covered unchanged.
1A1: correct
2M1: Double covered +e; one uncovered – e; and one single covered unchanged.
2A1ft: ft correct.
3A1ft: ft correct - no errors
(d) 1B1: cao.

June 2009 Decision Mathematics 2 6690/01

Question 3 notes:

(a) 1M1: Finds row maximin and column minimax. All six values enough, condone one slip.
1A1: Row maximin = -2 col minimax = 1 identified in some way.
2A1: Row maximin (-2) $\neq$ column minimax (1) stated and a clear link to statement.

(b) 1B1: cao, deletes column 3.

(c) 1B1ft: Matrix transposed **or** signs changed permit one error
2B1ft: cao

(d) 1M1: Setting up three probability equations, implicit definition of p.
1A1: cao
2M1: At least two lines correct, accept $p>1$ or $p<0$ here.
1A1: 3 lines cao, $0 \leq p \leq 1$, scale clear (or 1 line = 1), condone lack of labels.
2M1: ft their 3 lines. Setting up an equation in p, to find optimal and finding $0 \leq p \leq 1$.
2A1ft: p correct for their optimum. Options listed condone absence of dominated choice
3A1: cao

Question 6 notes:

(a) 1B1: cao.
(b) 1B1: cao.
(c) 1M1: A valid route ZA used as an empty square.
1A1 : $\theta = 9$, exiting square stated and correct.
2A1: Improved solution of exactly 5 numbers.
(d) 1M1: 6 shadow costs and 4 non-zero improvement indices stated
1A1: 6 shadow costs correct
2A1ft: precisely 4 improvement indices correct ft from their (c) and their shadow costs.
2M1: A valid route, negative II chosen, only one empty square used, θ's balance.
3A1ft: ft route, exiting square, entering square, θ value all correct
4A1: cao
(e) 1B1 cao

Question 7 notes:

(a) 1M1: Remaining four rows of stage 2 state 250 completed.
1A1: These four rows cao.
2A1; Stage 2 state 200 correct.
2M1: Completing at least 3 more states of stage 2. Something in every column.
3A1: Remainder of stage 2 state 150 completed correctly
4A1: Stage 2 states 100, 50 and 0 completed correctly.
3M1: Stage 3 completed
5A1: cao
1B1; Income correct including units
2B1: Investment allocation correct

(b) 1B1: Meaning of Stage correct. Generous
2B1: Meaning of State correct. Generous
3B1: Meaning of Action correct. Generous