**June 2006**
**6690 Decision Maths D2**
**Mark Scheme**

| Question Number | Scheme | Marks |
|---|---|---|
| 1) (a) | Any part of an optimal path is itself optimal | B1 |
| (b) | The route chosen such that the maximum arc length is as small as possible | B1 |
| (c) | e.g. maximising freight by minimising fuel needed when planning multiple stage light aircraft journey. | B2,1,0 [4] |

B1 cao ("part" "section" etc, "arc" "stage" "activity" "event" not)

B1 cao (not min of max route, not minimise longest arc)

B2 cao

B1 close "Bod" gets B1

| Question Number | Scheme | Marks |
|---|---|---|
| 2) | Let $x_{ij} = \begin{cases} 1 \text{ if worker does task} \\ 0 \text{ otherwise} \end{cases}$ | B1 |
| | where $x_{ij}$ indicates the arc from node $i$ to node $j$ $i \in P, Q, R$ $j \in 1, 2, 3$ | B1 (2) |
| | $x_{P1} + x_{P2} + x_{P3} = 1$ and $x_{P1} + x_{Q1} + x_{R1} = 1$ | M1 |
| | $x_{Q1} + x_{Q2} + x_{Q3} = 1$ $x_{P2} + x_{Q2} + x_{R2} = 1$ | A1 |
| | $x_{R1} + x_{R2} + x_{R3} = 1$ $x_{P3} + x_{Q3} + x_{R3} = 1$ | A1 (3) |
| | minimise, $C = 8x_{P1} + 7x_{P2} + 3x_{P3} + 9x_{Q1} + 5x_{Q2} + 6x_{Q3} + 10x_{R1} + 4x_{R2} + 4x_{R3}$ where $C$ is in hundreds of pounds | B1, B1 (2) [7] |

B1 cao

B1 defining variables – attempt

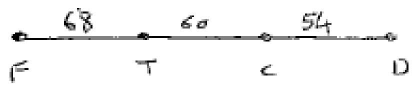
M1 At least 3 equations – coefficients of one

A1 cao 3 correct

A1 cao 6 correct

B1 Minimise

B1 cao (condone a slip) (– accept cost in pounds)

| | | |
|---|---|---|
| 3 (a) | Each activity must be visited once and then we return to the starting activity, this must be done in a minimum time | B2, 1, 0 (2) |
| (b) | $108 + 54 + 150 + 68 + 100 = 480$ minutes ($\doteq 8$ hours) | M1 A1 (2) |
| (c) | Use nearest neighbour B F T C D B<br>$64 + 68 + 60 + 54 + 150 = 396$ minutes ($< 7$ hours) | M1 A1<br>A1 (3) |
| (d) | <br>CT, TF, CD (Prim or Kruskal)<br>$182 + 64 + 100 = 346$ minutes | M1 A1<br>M1 A1√ (4)<br>[11] |

(a) B2 cao - all 3 bits in the context
B1 close 'Bod' is B1 (eg not in context; Just 'each activity once' - but not all 3; ....)
(b) M1 (maybe implicit) attempting to add 5 values
A1 cao
(c) M1 Each vertex visited once – either NN or 2×MST–shortcut (B0)
A1 cao incl return to B (BFTCDB)
A1 cao (396)
(d) M1 Finding correct ~~residual~~ minimum spanning tree (maybe implicit) 182 sufficient
A1 cao tree or 182
M1 adding 2 least arcs to B ie 100 and 64 only
A1√ cao √ from their m.s.t value ie 164 + their tree length.

4) (a) Adding $n \geq 20$ to table to give

| | H | P | R | W |
|---|---|---|---|---|
| A | 3 | 5 | 11 | 9 |
| B | 3 | 7 | 8 | n |
| C | 2 | 5 | 10 | 7 |
| D | 8 | 3 | 7 | 6 |

B1

Reducing rows first $\begin{bmatrix} 0 & 2 & 8 & 6 \\ 0 & 4 & 5 & n-3 \\ 0 & 3 & 8 & 5 \\ 5 & 0 & 4 & 3 \end{bmatrix}$ then columns $\begin{bmatrix} 0 & 2 & 4 & 3 \\ 0 & 4 & 1 & n-6 \\ 0 & 3 & 4 & 2 \\ 5 & 0 & 0 & 0 \end{bmatrix}$

M1 A1 (3)

either $\begin{bmatrix} 0 & 1 & 3 & 2 \\ 0 & 3 & 0 & n-7 \\ 0 & 2 & 3 & 1 \\ 6 & 0 & 0 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 & 3 & 2 \\ 0 & 3 & 0 & n-7 \\ 0 & 2 & 3 & 1 \\ 6 & 0 & 0 & 0 \end{bmatrix}$

M1 A1√

$\downarrow$ $\qquad\qquad$ $\downarrow$

$\begin{bmatrix} 0 & 0 & 2 & 1 \\ 1 & 3 & 0 & n-7 \\ 0 & 1 & 2 & 0 \\ 7 & 0 & 0 & 0 \end{bmatrix}$ $\qquad$ $\begin{bmatrix} 0 & 0 & 3 & 1 \\ 0 & 2 & 0 & n-8 \\ 0 & 1 & 3 & 0 \\ 7 & 0 & 1 & 0 \end{bmatrix}$

M1 A1√ (4)

A - H or P
B - R or R
C - W or H
D - P or W

cost £21 000

A1 A1 (2)

(b) Not unique - gives the other solution

M1 A1√ (2)

[11]

# D2 (6690) June 2006 Question 4

Doesn't replace - B0

Either (A)

Rows $\begin{bmatrix} 0 & 2 & 8 & 6 \\ 0 & 4 & 5 & - \\ 0 & 3 & 8 & 5 \\ 5 & 0 & 4 & 3 \end{bmatrix}$ columns $\begin{bmatrix} 0 & 2 & 4 & 3 \\ 0 & 4 & 1 & - \\ 0 & 3 & 4 & 2 \\ 5 & 0 & 0 & 0 \end{bmatrix}$ M1 A0 (3)

$$\begin{array}{c} \\ (c) \\ \\ \\ \end{array}\begin{bmatrix} 0 & 1 & 3 & 2 \\ 0 & 3 & 6 & - \\ 0 & 2 & 3 & 1 \\ 6 & 0 & 0 & 0 \end{bmatrix}\begin{array}{c} \\ (a) \\ \\ \\ \end{array}$$

M1 A0

(a) (b)

$\begin{bmatrix} 0 & 0 & 2 & 1 \\ 1 & 3 & 0 & - \\ 0 & 1 & 2 & 0 \\ 7 & 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 3 & 1 \\ 0 & 2 & 0 & - \\ 0 & 1 & 3 & 0 \\ 7 & 0 & 1 & 0 \end{bmatrix}$ M1 A0 (4)

A - H | P

B - R | or R | cost £2100

C - W | H

D - P | W

A1 A1 (2)

(b) not unique - gives other solution M1 A1 (2)

So must have 4 marks

7 max

## OR (B)

$$\text{Rows}\begin{bmatrix} 0 & 2 & 8 & 6 \\ 3 & 7 & 8 & - \\ 0 & 3 & 8 & 5 \\ 5 & 0 & 4 & 3 \end{bmatrix} \qquad \text{columns}\begin{bmatrix} 0 & 2 & 4 & 6 \\ 3 & 7 & 4 & - \\ 0 & 3 & 4 & 5 \\ 5 & 0 & 0 & 3 \end{bmatrix}$$

B0
M1
A0
(3)

$$\begin{bmatrix} 0 & 0 & 2 & 4 \\ 3 & 5 & 2 & - \\ 0 & 1 & 2 & 3 \\ 7 & 0 & 0 & 3 \end{bmatrix}$$

M1
A0

$$\begin{bmatrix} 0 & 0 & 2 & 4 \\ 1 & 3 & 0 & - \\ 0 & 1 & 2 & 3 \\ 7 & 0 & 0 & 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 2 & 4 \\ 3 & 4 & 1 & - \\ 0 & 0 & 1 & 2 \\ 8 & 0 & 0 & 3 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 & 2 \\ 3 & 5 & 0 & - \\ 0 & 1 & 0 & 1 \\ 9 & 2 & 0 & 3 \end{bmatrix}$$

M1
A0
(4)

All need more iterations

| | | | |
|---|---|---|---|
| A – | H | | P |
| B – | R | or | R |
| C – | W | | H |
| D – | P | | W |

cost £21 000

A1
A1
(2)

(b) Not unique – gives other solution  M1 A1✓ (2)

So must loose 4 marks

7 max

Q4

(a) B1 cao accept $\infty$ (but penalise $\infty - 1$ etc later - each time)

M1 Rows then columns

(3) A1 cao

M1 Double covered (2-3 lines) increased by 1, at least one single covered element unchanged, at least one uncovered - 1

A1✓cao ✓ on earlier error

M1 In at least 1 case (3-4 lines) double covered + 1, uncovered - 1 and single covered unchanged

(4) A1✓cao ✓ on earlier error

A1 cao must be listed

A1 cao including the zeros

(b) M1 A correct statement + reason accept indication on diagram

A1✓ other correct solution or carefully explained reason must be listed

If doesn't replace - B0

(a) Then may be eligible for M1 but A0

M1 but A0

M1 then 1st A0 but eligible for other two

(b) Eligible for M1 A1 So 7 max

See sheets

5)

| Stage | State | Action | Value |
|---|---|---|---|
| 1 | H | HT | 4 * |
| | I | IT | 3 * |
| | J | JT | 12 * |
| | K | KT | 20 * |
| 2 | D | DH | 2+4 = 6 |
| | | DI | 4+3 = 7 * |
| | E | EH | 3+4 = 7 * |
| | | EI | 4+3 = 7 * |
| | F | FJ | 10+12 = 22 * |
| | | FK | -8+20 = 12 |
| | G | GJ | 10+12 = 22 |
| | | GK | 17+20 = 37 * |
| 3 | A | AD | 3+7 = 10 |
| | | AE | 2+7 = 9 |
| | | AF | -5+22 = 17 * |
| | B | BD | 3+7 = 10 |
| | | BE | 2+7 = 9 |
| | | BF | -6+22 = 16 * |
| | C | CF | 8+22 = 30 * |
| | | CG | -15+37 = 22 |
| 4 | S | SA | 2+17 = 19 |
| | | SB | 3+16 = 19 |
| | | SC | -10+30 = 20 * |

Route S C F J T £20 000

M1 A1 (2)

M1 A1

A1 (3)

M1 A1 $\sqrt{}$

A1 $\sqrt{}$ (3)

M1 A1 $\sqrt{}$ (2)

M1 A1 (2)

$\boxed{12}$

M1 First stage completed
(2) A1 c.a.o (condone lack of *) Penalise silly states etc here and on last M mark
M1 Second stage completed - bod something in each column maximum only Penalise absence of stage or state column with removal of 1+2 A marks earned; ignore any * or always
A1 D + E states correct } penalise * one A1 only once
(3) A1 F + G states correct
M1 third stage completed - bod something in each column maximum only
A1√ 'A' state correct } penalise * one A1 only once (but may be 2nd time)
(3) A1√ B + C states correct
M1 Forth stage completed - bod maximum only
(2) A1√ c a o
M1 Must be able to √ from * + a route stated + a value stated
A1 c a o including zeros

## Special cases

SC1 minimum, minimax, maximin } 1st M1 A1 2nd M0 3rd M1 A0 A0 4th M1 A1√ 5th M1 A1√ $\boxed{7}$

SC2 maximax, minimin } 1st M1 A1 2nd M0 3rd M0 4th M0 5th M1 A1√ (if can be √) $\boxed{4}$

SC3 maximum working forwards - see sheet $\boxed{8}$

SC4 - if necessary SC1 but working forwards $\boxed{3}$ max ring SGJ!

D2 (6690) June 2006 Q5 working forwards

(5 ③)

| Stage | state | Action | value | |
|---|---|---|---|---|
| 1 | S | SA | 2 * | M0 |
| | | SB | 3 * | |
| | | SC | -10 * | |
| 2 | A | AD | 3 + 2 = 5 | M1 |
| | | AE | 2 + 2 = 4 | |
| | | AF | -5 + 2 = -3 | |
| | B | BD | 3 + 3 = 6 | |
| | | BE | 2 + 3 = 5 | A1 |
| | | BF | -6 + 3 = -3 | |
| | C | CF | 8 + -10 = -2 | A1 |
| | | CG | -15 + -10 = -25 | |
| 3 | D | DH | 2 + 6 = 8 | |
| | | DI | 4 + 6 = 10 | M1 |
| | E | EH | 3 + 5 = 8 | |
| | | EI | 4 + 5 = 9 | A1 |
| | F | FJ | 10 + -2 = 8 | |
| | | FK | -8 + -2 = -10 | A1 |
| | G | GJ | 10 + -25 = -15 | |
| | | GK | 17 + -25 = -8 | |
| 4 | H | HT | 4 + 8 = 12 | M1 |
| | I | IT | 3 + 10 = 13 | A1 |
| | J | JT | 12 + 8 = 20 * | |
| | K | KT | 20 + -8 = 12 | |

Route S C F J T M0

Value £20 000

(8 max)

6) (a) either e.s.
In an $n \times m$ problem, a degenerate solution occurs when the number of cells used is less than $(n+m-1)$ — B2, 1, 0 (2)

or e.s. when all the demand for one destination is satisfied by all the supply from a source, before the final demand and supplies are allocated.

(b) If total supply > total demand a dummy is used to absorb the excess — B1 (1)

(c)
$$\begin{bmatrix} 15 & & \\ 1 & 11 & 0 \\ & & 17 \end{bmatrix}$$
B1 (1)

(d) shadow costs $S_A = 0 \quad S_B = -1 \quad S_C = -1$

$D_1 = 62 \quad D_2 = 49 \quad D_3 = 1$ — M1

Improvement Indices
$I_{A2} = 47 - 0 - 49 = -2$ *
$I_{A3} = 0 - 0 - 1 = -1$
$I_{C1} = 68 + 1 - 62 = 7$
$I_{C2} = 58 + 1 - 49 = 10$

| | 1 (62) | 2 (49) | 3 (1) |
|---|---|---|---|
| (0) A | $15-\theta$ | $\theta$ | |
| (−1) B | $1+\theta$ | $11-\theta$ | 0 |
| (−1) C | | | 17 |

Entering A2, Exiting B2, $\theta = 11$

A1
A1√ (3)

shadow costs $S_A = 0 \quad S_B = -1 \quad S_C = -1$

$D_1 = 62 \quad D_2 = 47 \quad D_3 = 1$ — M1

Improvement Indices
$I_{A3} = 0 - 0 - 1 = -1$ *
$I_{B2} = 48 + 1 - 47 = 2$
$I_{C1} = 68 + 1 - 62 = 7$
$I_{C2} = 58 + 1 - 47 = 12$

| | 1 (62) | 2 (47) | 3 (1) |
|---|---|---|---|
| (0) A | $4-\theta$ | 11 | $\theta$ |
| (−1) B | $12+\theta$ | | $0-\theta$ |
| (−1) C | | | 17 |

Entering A3, Exiting B3, $\theta = 0$

A1
A1√ (3)

| | 1 (62) | 2 (47) | 3 (0) |
|---|---|---|---|
| (0) A | 4 | 11 | 0 |
| (−1) B | 12 | | |
| (0) C | | | 17 |

shadow costs $S_A = 0 \quad S_B = -1 \quad S_C = 0$

$D_1 = 62 \quad D_2 = 47 \quad D_3 = 0$ — M1 A1

Improvement Indices
$I_{B2} = 48 + 1 - 47 = 2$
$I_{B3} = 0 + 1 - 0 = 1$ — B1
$I_{C1} = 68 - 0 - 62 = 6$
$I_{C2} = 58 - 0 - 47 = 11$

$\therefore$ optimal (since all II $\geq 0$)

cost 1497 units — B1 (4)

$\boxed{14}$

Q6

(a) B2 CAO

B1 close "bed" is B1

(b) B1 cao must see > not ≠ o.e.

(c) B1 cao total of 5 numbers

(d) M1 shadow costs + at least 4 II's + stepping stone route stated + plausible (1 entering)

A1 shadow costs + 4 II's correct penalise excess values

(3) A1√ stepping stone route correct + θ correct + next solution (5 numbers) + penalise extra zeros here

M1 shadow costs + at least 4 II's + stepping stone route stated + plausible (1 entering)

A1 shadow costs + 4 II's correct penalise excess values +

(3) A1√ stepping stone route correct + θ correct + next solution (5 numbers) + penalise extra zeros here

M1 shadow costs + at least 4 II's stated.

A1 shadow costs + 4 II's correct

B1 conclusion (optimal) + solution stated (5 numbers)

B1 cao

Notes

(d) if $I_{A3}$ chosen

| | 1 | 2 | 3 |
|---|---|---|---|
| A | 15-θ | | θ |
| B | 1+θ | | 0-θ |
| C | | | |

Entering A3
Exiting B3
θ = 0

| | 1 (62) | 2 (49) | 3 (0) |
|---|---|---|---|
| 0 A | 15-θ | θ | 0 |
| (-1) B | 1+θ | 11-θ | |
| (0) C | | | 17 |

shadow costs $S_A = 0$ $S_B = -1$ $S_C = 0$

$D_1 = 62$ $D_2 = 49$ $D_3 = 0$

$I_{A2} = 47-0-49 = -2$ * $I_{C1} = 68-62-0 = 6$

$I_{B3} = 0+1-0 = 1$ $I_{C2} = 58-49-0 = 9$

Entering A2, Exiting B2 θ = 11

| | 1 (62) | 2 (47) | 3 (0) |
|---|---|---|---|
| (0) A | 4 | 11 | 0 |
| (-1) B | 12 | | |
| (0) C | | | 17 |

shadow cost $S_A = 0$ $S_B = -1$ $S_C = 0$

$D_1 = 62$ $D_2 = 47$ $D_3 = 0$

$I_{B2} = 48+1-47 = 2$ $I_{C1} = 68-62+0 = 6$

$I_{B3} = 0+1-0 = 1$ $I_{C2} = 58-47-0 = 11$

∴ optimal

7) (a) E.g. Maximise $P = V$ — B1

subject to $V - 5p_1 - 3p_2 - 6p_3 + r = 0$ — M1

$V - 7p_1 - 8p_2 - 4p_3 + s = 0$ — A2, 1, 0

$V - 2p_1 - 4p_2 - 9p_3 + t = 0$

$p_1 + p_2 + p_3 \, (+ \, u) = 1$

where $V$ = value of game to A, $p_i$ = probability of A playing row $i$ — B1

$p_i \geqslant 0$ and $r, s, t, u$ are slack variables all $\geqslant 0$ — (5)

(b) Not reducible and a 3 variable problem — B1 (1)

(c) e.g. — M1 A1 (2)

| b.v. | $V$ | $p_1$ | $p_2$ | $p_3$ | $r$ | $s$ | $t$ | $u$ | value |
|---|---|---|---|---|---|---|---|---|---|
| $r$ | 1 | −5 | −3 | −6 | 1 | 0 | 0 | 0 | 0 |
| $s$ | 1 | −7 | −8 | −4 | 0 | 1 | 0 | 0 | 0 |
| $t$ | 1 | −2 | −4 | −9 | 0 | 0 | 1 | 0 | 0 |
| $u$ | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| $P$ | −1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

M1 A1 A1 B1√ (4)

| b.v. | $V$ | $p_1$ | $p_2$ | $p_3$ | $r$ | $s$ | $t$ | $u$ | value | Row ops |
|---|---|---|---|---|---|---|---|---|---|---|
| $V$ | 1 | −5 | −3 | −6 | 1 | 0 | 0 | 0 | 0 | $R_1 \div 1$ |
| $s$ | 0 | −2 | −5 | 2 | −1 | 1 | 0 | 0 | 0 | $R_2 - R_1$ |
| $t$ | 0 | 3 | −1 | −3 | −1 | 0 | 1 | 0 | 0 | $R_3 - R_1$ |
| $u$ | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | $R_4$ stet |
| $P$ | 0 | −5 | −3 | −6 | 1 | 0 | 0 | 0 | 0 | $R_5 + R_1$ |

M1 A1√ A1 B1√ (4)

| b.v. | $V$ | $p_1$ | $p_2$ | $p_3$ | $r$ | $s$ | $t$ | $u$ | value | Row ops |
|---|---|---|---|---|---|---|---|---|---|---|
| $V$ | 1 | −11 | −18 | 0 | −2 | 3 | 0 | 0 | 0 | $R_1 + 6R_2$ |
| $p_3$ | 0 | −1 | $-\frac{5}{2}$ | 1 | $-\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 | 0 | $R_2 \div 2$ |
| $t$ | 0 | 0 | $-\frac{17}{2}$ | 0 | $-\frac{5}{2}$ | $\frac{3}{2}$ | 1 | 0 | 0 | $R_3 + 3R_2$ |
| $u$ | 0 | 2 | $\frac{7}{2}$ | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | 1 | 1 | $R_4 - R_2$ |
| $P$ | 0 | −11 | −18 | 0 | −2 | 3 | 0 | 0 | 0 | $R_5 + 6R_2$ |

$\boxed{16}$

Q 7

(a) B1 Maximise / minimise + consistent function

M1 Constraints (condone non-negativity) – at least 1 correct must be equations

A2 all correct

A1 at least 2 correct

B1 Defining variable

(b) B1 cao – both

(c) M1 Complete initialised tableau

(2) A1 cao

M1 Pivot correct + table complete ✓ on previous tableau

A1 Pivotal row correct incl b.v.

A1 cao

(4) B1✓ Row operations correctly stated (condone lack of pivot row operated)

M1 Pivot correct + table complete ✓ on previous tableau

A1✓ Pivotal row incl b.v.

A1 cao

(4) B1✓ Row operations correctly stated (condone lack of pivot row operation)

Note: If trying to minimise (c) M0 M0 B0 M0 B0

## Alt 1 – using $\frac{1}{v}$ argument

Let $p_1, p_2, p_3$ be probability of A playing 1, 2 and 3 respectively
Let $x_i = \frac{p_i}{v}$ B1

(a) minimise $P = x_1 + x_2 + x_3$

subject to: $5x_1 + 3x_2 + 6x_3 - r = 1$ M1

$7x_1 + 8x_2 + 4x_3 - s = 1$ A2,1,0

$2x_1 + 4x_2 + 9x_3 - t = 1$

$r, s, t, \ x_i \geq 0$ B1

(5)

(b) as scheme B1

(c) Can't solve for A – either need to solve for B – then use dual argument. (1)

– or transpose argument + matrix

(i) For B max $Q = y_1 + y_2 + y_3$

subject to $5y_1 + 7y_2 + 2y_3 + r_1 = 1$

$3y_1 + 8y_2 + 4y_3 + s_1 = 1$

$6y_1 + 4y_2 + 9y_3 + t_1 = 1$

$y_1, r_1, s_1, t_1 \geq 0$

| b.v. | $y_1$ | $y_2$ | $y_3$ | $r_1$ | $s_1$ | $t_1$ | value |
|---|---|---|---|---|---|---|---|
| $r_1$ | 5 | 7 | 2 | 1 | 0 | 0 | 1 |
| $s_1$ | 3 | 8 | 4 | 0 | 1 | 0 | 1 |
| $t_1$ | 6 | 4 | 9 | 0 | 0 | 1 | 1 |
| Q | −1 | −1 | −1 | 0 | 0 | 0 | 0 |

M1 A1 (2)

Alt 1 (c)(i)(a) choosing $y_1$ pivot first

eg

| b.v. | $y_1$ | $y_2$ | $y_3$ | $r_1$ | $s_1$ | $t_1$ | value | Row ops | |
|---|---|---|---|---|---|---|---|---|---|
| $r_1$ | 0 | $\frac{11}{3}$ | $-\frac{11}{2}$ | 1 | 0 | $-\frac{5}{6}$ | $\frac{1}{6}$ | $R_1 - 5R_3$ | M1 A1 |
| $s_1$ | 0 | 6 | $-\frac{1}{2}$ | 0 | 1 | $-\frac{1}{2}$ | $\frac{1}{2}$ | $R_2 - 3R_3$ | A1 |
| $y_1$ | 1 | $\frac{2}{3}$ | $\frac{3}{2}$ | 0 | 0 | $\frac{1}{6}$ | $\frac{1}{6}$ | $R_3 \div 6$ | B1√ |
| Q | 0 | $-\frac{1}{3}$ | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{6}$ | $\frac{1}{6}$ | $R_4 + R_3$ | (4) |

Alt 1 (c) (i) ctd – choosing $y_1$ pivot first

| b.v. | $y_1$ | $y_2$ | $y_3$ | $r_1$ | $s_1$ | $t_1$ | value | Row ops | |
|---|---|---|---|---|---|---|---|---|---|
| $y_2$ | 0 | 1 | $-\frac{3}{2}$ | $\frac{3}{11}$ | 0 | $-\frac{3}{22}$ | $\frac{1}{22}$ | $R_1 \div \frac{11}{3}$ | M1 A1√ |
| $s_1$ | 0 | 0 | $\frac{17}{2}$ | $-\frac{18}{11}$ | 1 | $\frac{19}{22}$ | $\frac{5}{22}$ | $R_2 - 6R_1$ | A1 |
| $y_1$ | 1 | 0 | $\frac{5}{2}$ | $-\frac{2}{11}$ | 0 | $\frac{7}{22}$ | $\frac{3}{22}$ | $R_3 - \frac{1}{3}R_1$ | B1√ |
| Q | 0 | 0 | 0 | $\frac{1}{11}$ | 0 | $\frac{1}{11}$ | $\frac{2}{11}$ | $R_4 + \frac{1}{3}R_1$ | (4) |

Alt 1(c)(ii)(b) choosing $y_2$ pivot first

| b.v. | $y_1$ | $y_2$ | $y_3$ | $r_1$ | $s_1$ | $t_1$ | value | Row ops | |
|---|---|---|---|---|---|---|---|---|---|
| r | $\frac{19}{8}$ | 0 | $-\frac{3}{2}$ | 1 | $-\frac{7}{8}$ | 0 | $\frac{1}{8}$ | $R_1 - 7R_2$ | M1 |
| $y_2$ | $\frac{3}{8}$ | 1 | $\frac{1}{2}$ | 0 | $\frac{1}{8}$ | 0 | $\frac{1}{8}$ | $R_2 \div 8$ | A1 |
| $t_1$ | $\frac{9}{2}$ | 0 | 7 | 0 | $-\frac{1}{2}$ | 1 | $\frac{1}{2}$ | $R_3 - 4R_2$ | A1 |
| Q | $-\frac{5}{8}$ | 0 | $-\frac{1}{2}$ | 0 | $\frac{1}{8}$ | 0 | $\frac{1}{8}$ | $R_4 + R_2$ | B1√ (4) |

| b.v. | $y_1$ | $y_2$ | $y_3$ | $r_1$ | $s_1$ | $t_1$ | value | Row ops | |
|---|---|---|---|---|---|---|---|---|---|
| $y_1$ | 1 | 0 | $-\frac{12}{19}$ | $\frac{8}{19}$ | $-\frac{7}{19}$ | 0 | $\frac{1}{19}$ | $R_1 \div \frac{19}{8}$ | M1 |
| $y_2$ | 0 | 1 | $\frac{14}{19}$ | $-\frac{3}{19}$ | $\frac{5}{19}$ | 0 | $\frac{2}{19}$ | $R_2 - \frac{3}{8}R_1$ | A1√ |
| $t_1$ | 0 | 0 | $\frac{187}{19}$ | $-\frac{36}{19}$ | $\frac{22}{19}$ | 1 | $\frac{5}{19}$ | $R_3 - \frac{9}{2}R_1$ | A1 |
| Q | 0 | 0 | $-\frac{17}{19}$ | $\frac{5}{8}$ | $-\frac{2}{19}$ | 0 | $\frac{3}{19}$ | $R_4 + \frac{5}{8}R_1$ | B1√ (4) |

Alt 1(c)(ii)(c) choosing $y_3$ pivot first

| b.v. | $y_1$ | $y_2$ | $y_3$ | $r_1$ | $s_1$ | $t_1$ | value | Row ops | |
|---|---|---|---|---|---|---|---|---|---|
| $r_1$ | $\frac{11}{3}$ | $\frac{55}{9}$ | 0 | 1 | 0 | $-\frac{2}{9}$ | $\frac{7}{9}$ | $R_1 - 2R_3$ | M1 |
| $s_1$ | $\frac{1}{3}$ | $\frac{56}{9}$ | 0 | 0 | 1 | $-\frac{4}{9}$ | $\frac{5}{9}$ | $R_2 - 4R_3$ | A1 |
| $y_3$ | $\frac{2}{3}$ | $\frac{4}{9}$ | 1 | 0 | 0 | $\frac{1}{9}$ | $\frac{1}{9}$ | $R_3 \div 9$ | A1 |
| Q | $-\frac{1}{3}$ | $-\frac{5}{9}$ | 0 | 0 | 0 | $\frac{1}{9}$ | $\frac{1}{9}$ | $R_4 + R_3$ | B1√ |

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Alt 1 (c) (i)(a) ctd

| b.v. | $y_1$ | $y_2$ | $y_3$ | $r_1$ | $s_1$ | $t_1$ | value | Row ops | |
|---|---|---|---|---|---|---|---|---|---|
| $r_1$ | $\frac{187}{56}$ | 0 | 0 | 1 | $\frac{-55}{56}$ | $\frac{2}{9}$ | $\frac{13}{56}$ | $R_1 - \frac{55}{9}R_2$ | M1 |
| $y_2$ | $\frac{3}{56}$ | 1 | 0 | 0 | $\frac{9}{56}$ | $-\frac{1}{14}$ | $\frac{5}{56}$ | $R_2 \div \frac{56}{9}$ | A1✓ |
| $y_3$ | $\frac{9}{14}$ | 0 | 1 | 0 | $-\frac{1}{14}$ | $\frac{1}{7}$ | $\frac{1}{14}$ | $R_3 - \frac{4}{9}R_2$ | ~~A1~~ |
| Q | $\frac{-17}{56}$ | 0 | 0 | 0 | $\frac{5}{56}$ | $\frac{1}{14}$ | $\frac{9}{56}$ | $R_4 + \frac{5}{9}R_2$ | B1✓ (4) |

Alt 1 (c) (ii)

(a) For B matrix becomes $\begin{pmatrix} -5 & -3 & -6 \\ -7 & -8 & -4 \\ -2 & -4 & -9 \end{pmatrix}$ adding 10 → $\begin{array}{c|ccc} & A1 & A2 & A3 \\ B1 & 5 & 7 & 4 \\ B2 & 3 & 2 & 6 \\ B3 & 8 & 6 & 1 \end{array}$

(9) B1

+ definitions B1

max "p" = $y_1 + y_2 + y_3$ M1

subject to $5y_1 + 7y_2 + 4y_3 + r_1 = 1$ A2,1,0

$3y_1 + 2y_2 + 6y_3 + s_1 = 1$

$8y_1 + 6y_2 + y_3 + t_1 = 1$

$y_1, y_2, y_3, r_1, s_1, t_1 \geq 0$

(b) as scheme B1

(c)

| b.v. | $y_1$ | $y_2$ | $y_3$ | $r_1$ | $s_1$ | $t_1$ | value | |
|---|---|---|---|---|---|---|---|---|
| $r_1$ | 5 | 7 | 4 | 1 | 0 | 0 | 1 | M1 |
| $s_1$ | 3 | 2 | 6 | 0 | 1 | 0 | 1 | A1 |
| $t_1$ | 8 | 6 | 1 | 0 | 0 | 1 | 1 | (2) |
| p | -1 | -1 | -1 | 0 | 0 | 0 | 0 | |

(ii)(a) e.g. chooses 8 in $y_1$

| b.v. | $y_1$ | $y_2$ | $y_3$ | $r_1$ | $s_1$ | $t_1$ | value | Row ops | M1 |
|---|---|---|---|---|---|---|---|---|---|
| $r_1$ | 0 | $\frac{13}{4}$ | $\frac{27}{8}$ | 1 | 0 | $\frac{-5}{8}$ | $\frac{3}{8}$ | $R_1 - 5R_3$ | A1 |
| $s_1$ | 0 | $-\frac{1}{4}$ | $\frac{45}{8}$ | 0 | 1 | $-\frac{3}{8}$ | $\frac{5}{8}$ | $R_2 - 3R_3$ | A1 |
| $y_1$ | 1 | $\frac{3}{4}$ | $\frac{1}{8}$ | 0 | 0 | $\frac{1}{8}$ | $\frac{1}{8}$ | $R_3 \div 8$ | B1✓ |
| p | 0 | $-\frac{1}{4}$ | $-\frac{7}{8}$ | 0 | 0 | $\frac{1}{8}$ | $\frac{1}{8}$ | $R_4 + R_3$ | (4) |

## Alt 1 (c) (ii)(a) cont'd

either selects $\frac{27}{8}$ as pivot in which case tableau becomes (or $\frac{45}{8}$)

| b.v. | $y_1$ | $y_2$ | $y_3$ | $r_1$ | $s_1$ | $t_1$ | value | Row ops | |
|---|---|---|---|---|---|---|---|---|---|
| (r) $y_3$ | 0 (0) | $\frac{26}{27}$ ($\frac{17}{5}$) | 1 (0) | $\frac{8}{27}$ (1) | ($-\frac{3}{5}$) 0 | $-\frac{5}{27}$ ($-\frac{1}{5}$) | $\frac{1}{9}$ (0) | $R_1 \div \frac{27}{8}$ ($\frac{45}{8}$) | M1 |
| ($y_2$) $s_1$ | 0 (0) | $-\frac{17}{3}$ ($-\frac{8}{45}$) | 0 (1) | $-\frac{5}{3}$ (0) | ($\frac{8}{45}$) 1 | $\frac{2}{3}$ ($-\frac{4}{15}$) | 0 ($\frac{1}{9}$) | $R_2 - \frac{45}{8}R_1$ | A1✓ |
| ($y_1$) $y_1$ | 1 (1) | $\frac{17}{27}$ ($\frac{26}{45}$) | 0 (0) | $-\frac{1}{27}$ (0) | ($-\frac{17}{45}$) 0 | $\frac{4}{27}$ ($\frac{5}{15}$) | $\frac{1}{9}$ ($\frac{1}{9}$) | $R_3 - \frac{1}{8}R_1$ | A1 |
| P | 0 (0) | $\frac{16}{27}$ ($-\frac{13}{45}$) | 0 (0) | $\frac{7}{27}$ (0) | ($\frac{3}{45}$) 0 | $-\frac{1}{27}$ ($\frac{1}{15}$) | $\frac{2}{9}$ ($\frac{2}{9}$) | $R_4 + \frac{7}{8}R_1$ | B1✓ (4) |

### 1(c)(ii)(b) Selects 7 in $y_2$

| b.v. | $y_1$ | $y_2$ | $y_3$ | $r_1$ | $s_1$ | $t_1$ | value | Row ops | |
|---|---|---|---|---|---|---|---|---|---|
| $y_2$ | $\frac{5}{7}$ | 1 | $\frac{4}{7}$ | $\frac{1}{7}$ | 0 | 0 | $\frac{1}{7}$ | $R_1 \div 7$ | M1 |
| $s_1$ | $\frac{11}{7}$ | 0 | $\frac{13}{7}$ | $-\frac{2}{7}$ | 1 | 0 | $\frac{5}{7}$ | $R_2 - 2R_1$ | A1 |
| $t_1$ | $\frac{26}{7}$ | 0 | $-\frac{17}{7}$ | $-\frac{6}{7}$ | 0 | 1 | $\frac{1}{7}$ | $R_3 - 6R_1$ | A1 |
| P | $-\frac{2}{7}$ | 0 | $-\frac{3}{7}$ | $\frac{1}{7}$ | 0 | 0 | $\frac{1}{7}$ | $R_4 + R_1$ | B1✓ (4) |

| b.v. | $y_1$ | $y_2$ | $y_3$ | $r_1$ | $s_1$ | $t_1$ | value | Row ops | |
|---|---|---|---|---|---|---|---|---|---|
| $y_3$ | $\frac{5}{4}$ | $\frac{7}{4}$ | 1 | $\frac{1}{4}$ | 0 | 0 | $\frac{1}{4}$ | $R_1 \div \frac{4}{7}$ | M1 |
| $s_1$ | $-\frac{3}{4}$ | $-\frac{13}{4}$ | 0 | $-\frac{3}{4}$ | 1 | 0 | $\frac{1}{4}$ | $R_2 - \frac{13}{7}R_1$ | A1✓ |
| $t_1$ | $\frac{27}{4}$ | $\frac{17}{4}$ | 0 | $-\frac{1}{4}$ | 0 | 1 | $\frac{3}{4}$ | $R_3 + \frac{17}{7}R_1$ | A1 |
| P | $\frac{1}{4}$ | $\frac{3}{4}$ | 0 | $\frac{1}{4}$ | 0 | 0 | $\frac{1}{4}$ | $R_4 + \frac{3}{7}R_1$ | B1✓ (4) |

### 1(c)(ii)(c) Selects 6 in $y_3$

| b.v. | $y_1$ | $y_2$ | $y_3$ | r | s | t | value | Row ops | |
|---|---|---|---|---|---|---|---|---|---|
| $r_1$ | 4 | $\frac{17}{3}$ | 0 | 1 | $-\frac{2}{3}$ | 0 | $\frac{1}{3}$ | $R_1 - 4R_2$ | 0.33 M1 |
| $y_3$ | $\frac{1}{2}$ | $\frac{1}{3}$ | 1 | 0 | $\frac{1}{6}$ | 0 | $\frac{1}{6}$ | $R_2 \div 6$ | 0.5 A1 |
| $t_1$ | $\frac{15}{2}$ | $\frac{17}{3}$ | 0 | 0 | $-\frac{1}{6}$ | 1 | $\frac{5}{6}$ | $R_3 - R_2$ | 0.43 A1 |
| P | $-\frac{1}{2}$ | $-\frac{2}{3}$ | 0 | 0 | $\frac{1}{6}$ | 0 | $\frac{1}{6}$ | $R_4 + R_2$ | B1✓ (4) |

| b.v. | $y_1$ | $y_2$ | $y_3$ | r | s | t | value | Row ops | |
|---|---|---|---|---|---|---|---|---|---|
| $y_2$ | $\frac{12}{17}$ | 1 | 0 | $\frac{3}{17}$ | $-\frac{2}{17}$ | 0 | $\frac{1}{17}$ | $R_1 \div \frac{17}{3}$ | M1 |
| $y_3$ | $\frac{9}{34}$ | 0 | 1 | $-\frac{1}{17}$ | $\frac{7}{34}$ | 0 | $\frac{5}{34}$ | $R_2 - \frac{1}{3}R_1$ | A1✓ |
| $t_1$ | $\frac{7}{2}$ | 0 | 0 | $-1$ | $\frac{1}{2}$ | 1 | $\frac{1}{2}$ | $R_3 - \frac{17}{3}R_1$ | A1 B1✓ (4) |
| P | $-\frac{1}{34}$ | 0 | 0 | $\frac{2}{17}$ | $\frac{3}{34}$ | 0 | $\frac{7}{34}$ | $R_4 + \frac{1}{3}R_1$ | |

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Alt 2 – same as scheme, but eliminates $p_3$ using $p_3 = 1 - p_1 - p_2$

(a) maximise $P = V$ B1

subject to $V + p_1 + 3p_2 + r = 6$ M1

$V - 3p_1 - 4p_2 + s = 4$ A2, 1, 0

$V + 7p_1 + 5p_2 + t = 9$

$p_1, p_2, r, s, t, \geq 0$, where $p_i$ = probability of playing row $i$, $V$ = value of game B1 (…

(b) As scheme B1 (…

| b.v. | v | $p_1$ | $p_2$ | r | s | t | value | |
|---|---|---|---|---|---|---|---|---|
| r | 1 | 1 | 3 | 1 | 0 | 0 | 6 | M1 |
| s | 1 | -3 | -4 | 0 | 1 | 0 | 4 | A1 |
| t | 1 | 7 | 5 | 0 | 0 | 1 | 9 | (2) |
| P | -1 | 0 | 0 | 0 | 0 | 0 | 0 | |

| b.v. | v | $p_1$ | $p_2$ | r | s | t | value | Row ops | |
|---|---|---|---|---|---|---|---|---|---|
| r | 0 | 4 | 7 | 1 | -1 | 0 | 2 | $R_1 - R_2$ | M1 A1 |
| v | 1 | -3 | -4 | 0 | 1 | 0 | 4 | $R_2 \div 1$ | A1 |
| t | 0 | 10 | 9 | 0 | -1 | 1 | 5 | $R_3 - R_2$ | B1✓ |
| P | 0 | -3 | -4 | 0 | 0 | 0 | 4 | $R_4 + R_2$ | (4) |

| b.v. | v | $p_1$ | $p_2$ | r | s | t | value | Row ops | |
|---|---|---|---|---|---|---|---|---|---|
| $p_2$ | 0 | $\frac{4}{7}$ | 1 | $\frac{1}{7}$ | $-\frac{1}{7}$ | 0 | $\frac{2}{7}$ | $R_1 \div 7$ | M1 A1 |
| v | 1 | $-\frac{5}{7}$ | 0 | $\frac{4}{7}$ | $\frac{3}{7}$ | 0 | $\frac{36}{7}$ | $R_2 + 4R_1$ | A1 |
| t | 0 | $\frac{34}{7}$ | 0 | $-\frac{9}{7}$ | $\frac{16}{7}$ | 1 | $\frac{17}{7}$ | $R_3 - 9R_1$ | B1✓ |
| P | 0 | $-\frac{5}{7}$ | 0 | $\frac{4}{7}$ | $\frac{3}{7}$ | 0 | $\frac{36}{7}$ | $R_4 + 4R_1$ | (4) |