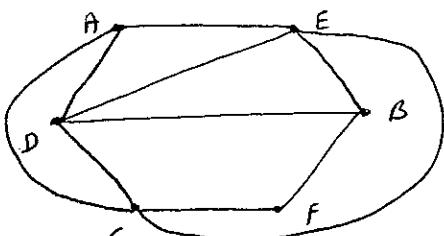


June 2005
6689 Decision D1
Mark Scheme

Question Number	Scheme	Marks
1)	e.g. 74 28 63 54 54 49 37 68 74 63 54 68 54 28 49 37 74 63 68 54 49 28 37 74 68 63 54 37 28 74 68 sort complete ∴ Ali, Sophie, Eun-Jung, {Katie + Marciana}, Peter, Rory, Bobby	m1 A1 A1 A1 (4) A1 (1) S
2)(a)	e.g. A E B F C D A	m1 A1 (2)
(b)	e.g. 	m1 A1 A1 (3)
(c)	State that one of these arcs (AF or EF) [Named], crosses at least one arc in each set. [Named arcs]	B2, 1/0 (2) T
3)(a)	$AC + DF = 8 + 9 = 17$ $AD + CF = 15 + 16 = 31$ $AF + CD = 13 + 7 = 20$ length = $77 + 17 = 94 \text{ km}$	m1 A1 A1 (3) m1 A1 (2)
(b)	shortest arc is CD (7) so use A and F as end points	B2, 1, 0 (2) T

4) (a)	<p>e.g.</p>	m1 A1 A1 A1 (4)
(b)	<p>Reference to K, J, G and L - K depends on J and G, but L depends on G only Both M and N must be uniquely represented in terms of events.</p>	B2, 1, 0 B1 (3) [17]
5) (a)	$E - 4 = B - 2 = D - 1 = A - 3 = C - 5$ change states to give matching $A = 3 \quad B = 2 \quad C = 5 \quad D = 1 \quad E = 4$ $E - 4 = B - 2 = D - 3 = C - 5$ change states to give matching $A = 1 \quad B = 2 \quad C = 5 \quad D = 3 \quad E = 4$	m1 A1 A1 (3) m1 A1 A1 (3)
(b)	<p>e.g. Reference to B + E and 4 + 2</p>	B2, 1, 0 (2) [8]
6) (a)	<p>Route: A C F E G J length: 53 km</p>	m1 A1 A1 ✓ A1 ✓ A1 A1 (3)
(b)	<p>General explanation - Trace back from J - Include arc xy if y is already on path and if difference in final labels equals length of arc.</p> <p>Specific explanation -</p> <ul style="list-style-type: none"> $53 - 15 = 38 \quad G-J$ $38 - 6 = 32 \quad E-G$ $32 - 4 = 28 \quad F-E$ $28 - 10 = 18 \quad C-F$ $18 - 18 = 0 \quad A-C$ 	B2/1/0 (2)
(c)	<p>e.g. ADFEGJ or ACEGJ ; length 54 km</p>	m1 A1; A1 (3) [10]

7) (a)	r, s and t are unused amounts of bird seed (in kg), suet blocks and peanuts (in kg) that Polly has at the end of each week after she has made up and sold her packs.	B2, 1, 0 (2)																																								
(b)	<table border="1" style="display: inline-table; vertical-align: middle;"> <thead> <tr> <th>b.v.</th> <th>x</th> <th>y</th> <th>z</th> <th>r</th> <th>s</th> <th>t</th> <th>value</th> </tr> </thead> <tbody> <tr> <td>z</td> <td>$\frac{2}{5}$</td> <td>$\frac{1}{2}$</td> <td>1</td> <td>$\frac{1}{10}$</td> <td>0</td> <td>0</td> <td>14</td> </tr> <tr> <td>s</td> <td>$\frac{2}{5}$</td> <td>-1</td> <td>0</td> <td>$-\frac{2}{5}$</td> <td>1</td> <td>0</td> <td>4</td> </tr> <tr> <td>t</td> <td>$-\frac{1}{5}$</td> <td>$\frac{1}{2}$</td> <td>0</td> <td>$-\frac{3}{10}$</td> <td>0</td> <td>1</td> <td>18</td> </tr> <tr> <td>P</td> <td>-90</td> <td>-25</td> <td>0</td> <td>65</td> <td>0</td> <td>0</td> <td>9100</td> </tr> </tbody> </table> <p> $R_1 \div 10$ $R_2 - 4R_1$ $R_3 - 3R_1$ $R_4 + 650R_1$ </p>	b.v.	x	y	z	r	s	t	value	z	$\frac{2}{5}$	$\frac{1}{2}$	1	$\frac{1}{10}$	0	0	14	s	$\frac{2}{5}$	-1	0	$-\frac{2}{5}$	1	0	4	t	$-\frac{1}{5}$	$\frac{1}{2}$	0	$-\frac{3}{10}$	0	1	18	P	-90	-25	0	65	0	0	9100	m1 A1 m1 A2✓, 1✓, 0 (5)
b.v.	x	y	z	r	s	t	value																																			
z	$\frac{2}{5}$	$\frac{1}{2}$	1	$\frac{1}{10}$	0	0	14																																			
s	$\frac{2}{5}$	-1	0	$-\frac{2}{5}$	1	0	4																																			
t	$-\frac{1}{5}$	$\frac{1}{2}$	0	$-\frac{3}{10}$	0	1	18																																			
P	-90	-25	0	65	0	0	9100																																			
(c)	$x = 0 \quad y = 0 \quad z = 14 \quad r = 0 \quad s = 4 \quad t = 18 \quad P = £91$	m1 A2✓, 1✓, 0 (3)																																								
(d)	$P = 90x + 25y + 65r = 9100 \quad (\text{o.e.})$	m1 A1✓																																								
(e)	$P = 9100 + 90x + 25y - 65r$ so increasing x or y would increase the profit	③ (B) ✓ (3)																																								
(f)	The $\frac{2}{5}$ in the x column and 2 nd (s) row.	B2✓, 1✓, 0 (2)																																								

8 (a) $SS_1 = 47$, $SS_2 = 87$, $T_1 T = 51$, $T_2 T = 73$ added to diagram 1

M1 A1 (2)

(b) $SS_1 \xrightarrow{47} S$, $SS_2 \xleftarrow{49} S$, $T_1 T \xrightarrow{8} T$, $T_2 T \xleftarrow{53} T$

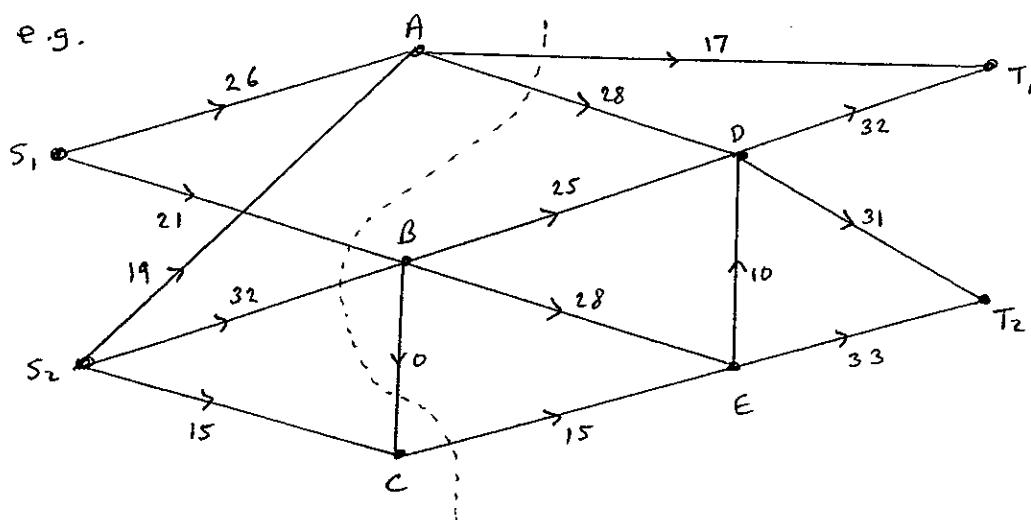
M1 A1 (2)

(c) e.g. $SS_1 A D T_1 T = 2$
 $SS_1 C E T_2 T = 1$
 $SS_2 C E D T_2 T = 10$
 $SS_2 C E B D T_1 T = 4$

M1
A4, 3, 2, 1, 0

maximum flow = 113

(d)



(B1) 16

M1 A1 (2)

(e) max flow-min cut theorem; cut AT₁, AD, S₁B, S₂B, BC, CE

(M1) A1 (2)

(f) Idea of a directed flow along arcs; from S to T; through a system; practical network

B2, 1, 0 (2)

16

Question Number	Scheme	Marks
1)	e.g. 74 28 63 54 (54) 49 37 68 74 63 (54) 68 (54) 28 (49) 37 74 (63) 68 (54) (49) 28 (37) 74 (68) (63) (37) (28) (74) (68) (28) 74 68 63 54 54 49 37 28 sort complete \therefore Ali, Sophie, Eun-Jung, {Katie + Marciana}, Peter, Rorry, Bobby	54 54 49 63 37 68 (28) A 1 (4) A 1 (1) [5]

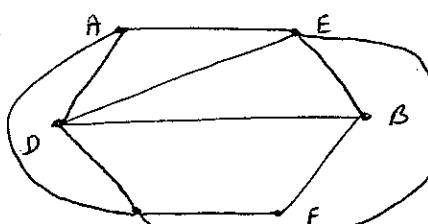
Q1 M1 Pivot clear list $> \boxed{P} >$. Bubble sort etc. M0

A1 1st pass correct, next pivot correctly selected consistently

A1^v 2nd + 3rd passes correct, pivot for next pass selected consistently each time. Penalise fragmented list here
(or list rewritten or all chosen as pivots)

A1 C.S.O. + stop statement (o.e.). Penalise non-sig no. errors here. Penalise "sloppiness" here

A1 C.A.O. accept c.a. even if m.s.

2)(a)	e.g. A E B F C D A	M1 A1 (2)
(b)	e.g. 	M1 A1 A1 (3)
(c)	States that one of these arcs (AF or EF) [Named], crosses at least one arc in each set. [Named arcs] written and	B2, 1/0 (2) [7]

Q2(a) M1 Each letter present exactly once - apart from possibly start + finish vertex

A1 a correct route - starts and finishes at A

(b) M1 Cycle drawn as hexagon + at least 1 other arc added to diagram

A1 at least 2 arcs added to hexagon

A1 C.A.O.

(c) B2 Good explanation AF or EF crosses named "inside" arc + named "outside" arc.

B1^v AF or EF crosses named arc. "close". 'bad' gets B1. If 1 crossing visible on graph give bad

G 1 Alternative correct answers

(i)	74 28 63 54 <u>54</u> 49 37 68 74 <u>63</u> 68 <u>54</u> 28 54 <u>49</u> 37 74 <u>68</u> <u>63</u> <u>54</u> <u>49</u> 28 <u>37</u> <u>74</u> <u>68</u> <u>54</u> <u>37</u> <u>28</u>	54 63 49 68 37 (54) A1✓	m1 A1
-----	--	----------------------------------	----------

(ii)	74 28 63 <u>54</u> 54 49 37 68 74 <u>63</u> 54 68 <u>54</u> 28 <u>49</u> 37 74 68 <u>63</u> <u>54</u> <u>49</u> <u>28</u> 37 <u>74</u> <u>68</u> <u>54</u> <u>37</u> <u>28</u>	54 63 49 74 28 (54) A1✓	m1 A1
------	---	----------------------------------	----------

(iii)	74 28 63 <u>54</u> 54 49 37 68 74 <u>63</u> 68 <u>54</u> 28 <u>54</u> 49 37 74 68 <u>63</u> <u>54</u> 28 <u>49</u> 37 <u>74</u> <u>68</u> <u>49</u> <u>28</u> 37 <u>37</u> <u>28</u>	54 63, 54 74, 49 28 (68) (37) A1✓	m1 A1
-------	--	--	----------

Put in list

(iv)	74 28 63 54 54 49 37 68 74 <u>28</u> 63 54 54 49 37 68 74 <u>63</u> 54 54 49 37 68 <u>28</u> 74 <u>68</u> <u>63</u> <u>54</u> 54 49 37 74 <u>68</u> <u>54</u> <u>54</u> 49 37 74 <u>68</u> <u>54</u> <u>54</u> 49 37 74 <u>68</u> <u>54</u> <u>49</u> 37 74 <u>68</u> <u>49</u> <u>37</u>	74 28 63 (68) 54 54 49 (37) A1	m1 A1✓
------	--	---	-----------

Ali, Sophie, Eun-Jing, Kate+Moriana, Peter, Rony, Bobby

Q1 MISREADS

-2 for MR

(MR)

(a)	74	28	63	54	54	49	37	68	54	m1
	28	54	49	37	54	74	63	68	49	A1
	28	37	49	54		63	74	68	37	68 (54)
	28	37		54			68	74		A1✓

(b)	74	28	63	54	54	49	37	68	54	m1
	28	49	37	54	74	63	54	68	49, 54	A1
	28	37	49		54	74	63	68	37, 63	
	28	37			63	74	68		68 (28)	A1✓

(c)	74	28	63	54	54	49	37	68	54	m1
	28	54	49	37	54	74	63	68	54, 63	A1
	28	49	37	54		63	74	68	49 74	
	28	37	49			68	74		28	
	28	37								A1✓

(d)	74	28	63	54	54	49	37	68	54	m1
	28	49	37	54	74	63	54	68	49 63	A1
	28	37	49		54	63	74	68	28, 74, (54)	
	28	37			54		68	74		A1✓

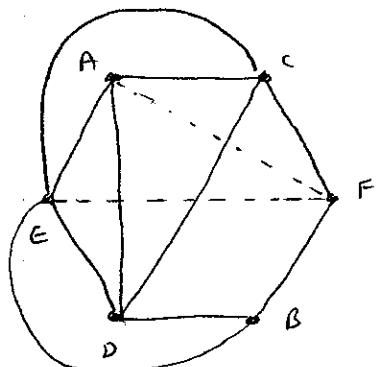
If candidates reverse list then restore full marks.
names or numbers

Bobby, Rony, Peter, Kate + Mariana, Eun-Jung, Sophie, Ali

Q2 Some e.g. Hamiltonian cycles + diagrams for Q2

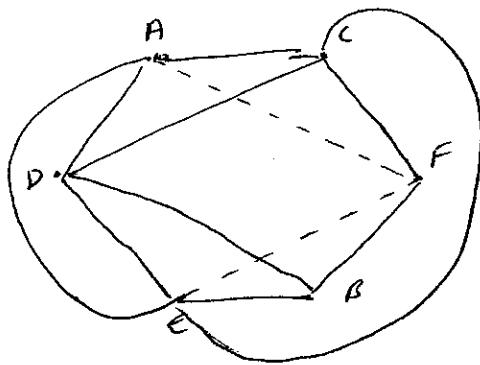
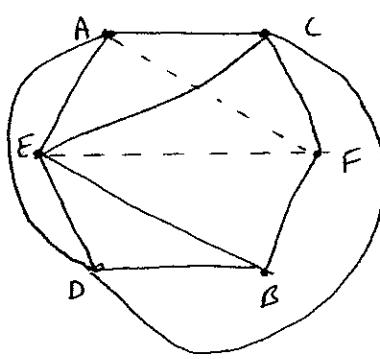
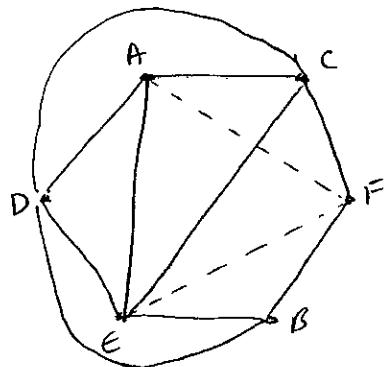
A C F B D E A

A E D B F C A



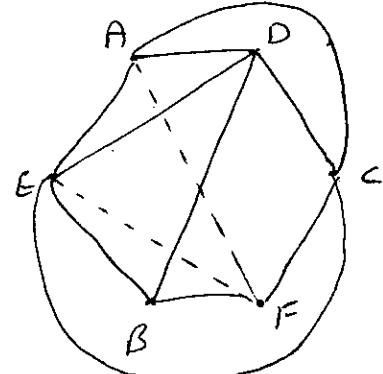
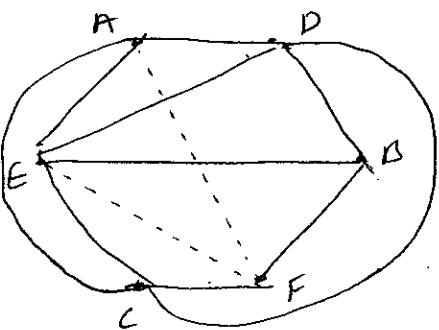
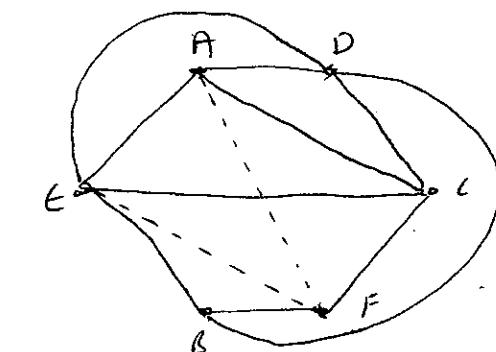
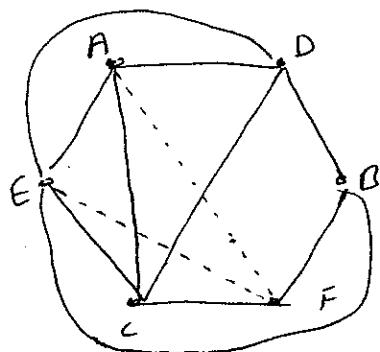
A C F B E D A

A D E B F C A



A D B F C E A

A E C F B D A



5) (a)	$E - 4 = B - 2 = D - 1 = A - 3 = C - 5$ change states to give matching $A = 3 \ B = 2 \ C = 5 \ D = 1 \ E = 4$	m1 A1 A1 (3)
	$E - 4 = B - 2 = D - 3 = C - 5$ change states to give matching $A = 1 \ B = 2 \ C = 5 \ D = 3 \ E = 4$	m1 A1 A1 (3)
(b)	e.g. Reference to B+E and 4+2	B2, 3, 0 (2) [8]

Q5 (a) m1 1st path E to 5

A1 c.a.o + c.s.

A1 matching c.a.o must be clear, must ✓

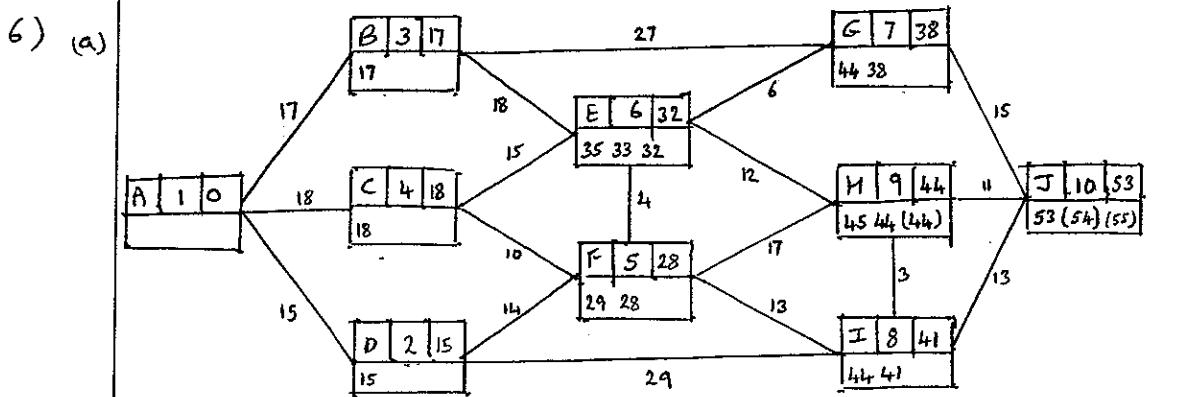
m1 2nd path E to 5

A1 c.a.o. + c.s. (^{c.s.} I don't penalise twice)

A1 matching c.a.o. must be clear, must ✓

(b) B2 Full clear explanation B, E, 2 and 4 listed. (+D) O.E ~ lots of alternatives

B1 Probably 3 out of 4 referred to, may be explanation confused, Superfluous filtering introduced. "b.o.d gets B1"



Route: A C F E G J

length: 53 km

M1
A1
A1 ✓
A1 ✓
A1

(b)

General explanation - Trace back from J

- Include arc XY if Y is already on path and if difference in final labels equals length of arc.

Specific explanation - $53 - 15 = 38$ GJ
 $38 - 6 = 32$ EG
 $32 - 4 = 28$ FE
 $28 - 10 = 18$ CF
 $18 - 18 = 0$ AC

B2/1/0 (2)

(c)

e.g. A D F E G J or A C E G J ; length 54 km

M1 A1; A1 (3)

10

6 (a) M1 In E or F or G or H or I w.v. large replaced by small . .

A1 A, B, C, D, F correct (order in rising sequence)

A1 ✓ E G I correct + labelling order (penalise order of labelling only once)

A1 ✓ H, J correct + labelling (penalise order of labelling only once)

A1 Route + length (both) condone lack of km.

(b) B2 ✓ Complete version of one of the 2 given explanations

B1 ✓ All the bar one step. 'bad' gets B1

(c) M1 Route A to J avoiding CF

A1 c.a.o or a description

(A1) 54 (condone lack of km)

7) (a) r , s and t are unused amounts of bird seed (in kg), suet blocks and peanuts (in kg) that Polly has at the end of each week after she has made up and sold her packs.

B2, 1, 0
(2)

(b)

b.v.	x	y	z	r	s	t	value
Z	$\frac{2}{5}$	$\frac{1}{2}$	1	$\frac{1}{10}$	0	0	14
S	$\frac{2}{5}$	-1	0	$-\frac{2}{5}$	1	0	4
T	$-\frac{1}{5}$	$\frac{1}{2}$	0	$-\frac{3}{10}$	0	1	18
P	-90	-25	0	65	0	0	9100

$$\begin{aligned} R_1 &\div 10 \\ R_2 &\leftarrow -4R_1 \\ R_3 &\leftarrow 3R_1 \\ R_4 &\leftarrow 650R_1 \end{aligned}$$

M1 A1
A2, 1, 0
(5)

(c) $x = 0 \quad y = 0 \quad z = 14 \quad r = 0 \quad s = 4 \quad t = 18 \quad P = £91$

M1
A2, 1, 0
(3)

(d) $P = 90x + 25y + 65r = 9100 \quad (\text{o.e.})$

M1 A1 ✓

(e) $P = 9100 + 90x + 25y - 65r$

so increasing x or y would increase the profit

(B) ✓ (3)

(f) The $\frac{2}{5}$ in the x column and 2nd (s) row.

B2, 1, 0
(2)

15

7) (a) B2 Ref to "unused" & bird seed, suet blocks & peanuts".

B1 Ref to "unused" or bird seed etc or muddled explanation. "bad" sets B1 must engage with context

(b) M1 correct pivot chosen

A1 pivot row correct c.a.o. incl b.v.

M1 ✓ correct row operations used (all 3) - at least 1 non zero or 1 term correct in each row. Whole row ✓ \Rightarrow M0

A2 ✓ non-pivoted rows correct; -1 each error ✓ on error in pivot ^{choice} only. Penalties b.v. once only

(c) M1 3 variables stated - must have completed b.v. + value columns (or 1's and zeros) on tableau. Any negative M0

A2 ✓ all 7 c.a.o. Need £91 ✓ but accept 9100

A1 ✓ at least 4 c.a.o. (condone $P = 9100, 1$)

(d) M1 ✓ P , $-90x$, $-25y$, $65r$ and 9100 (or 91) all present and one = sign

A1 ✓ c.a.o. (o.e.)

(e) (B) ✓ stating that increasing ~~x or y~~ would increase profit, probably re-arranging profit equation. Generous.

(f) B2 ✓ $\frac{2}{5}$ identified, x column and 2nd (s) row. Accept raised in last tableau

B1 ✓ 'bad' sets B1. If w/t from their "optimal" tableau B1,

Q 7 (b) notes

1) Wrong pivot chosen in col z. (- usually 4) Then form A2✓

(a)	b.v.	x	y	z	r	s	t	value	
(choose)	r	-1	$2\frac{1}{2}$	0	1	$-2\frac{1}{2}$	0	-10	$R_1 - 10R_2$
	z	$\frac{1}{2}$	$\frac{1}{4}$	1	0	$\frac{1}{4}$	0	15	$R_2 \div 4$
	E	$-\frac{1}{2}$	$\frac{1}{4}$	0	0	$-\frac{3}{4}$	1	15	$R_3 - 3R_2$
	P	-25	$-18\frac{1}{2}$	0	0	$162\frac{1}{2}$	0	9750	$R_4 + 650R_2$

(b)	b.v.	x	y	z	r	s	t	value	
(choose)	r	$\frac{2}{3}$	$-1\frac{2}{3}$	0	1	0	$-\frac{10}{3}$	-60	$R_1 - 10R_3$
	s	$\frac{2}{3}$	$-1\frac{2}{3}$	0	0	1	$-\frac{4}{3}$	-20	$R_2 - 4R_3$
	z	$\frac{1}{3}$	$\frac{2}{3}$	1	0	0	$\frac{1}{3}$	20	$R_3 \div 3$
	P	$-133\frac{1}{3}$	$83\frac{1}{3}$	0	0	0	$216\frac{2}{3}$	13000	$R_4 + 650R_3$

2) MISREADS - use col x or col y - 2 A marks if correct.

(a)	b.v.	x	y	z	r	s	t	value	
	r	0	3	2	1	-2	0	20	$R_1 - 4R_2$
	x	1	$\frac{1}{2}$	2	0	$\frac{1}{2}$	0	30	$R_2 \div 2$
	E	0	$1\frac{1}{2}$	1	0	$-\frac{1}{2}$	1	30	$R_3 - R_2$
	P	0	-175	50	0	175	0	10500	$R_4 + 350R_2$

(b)	b.v.	x	y	z	r	s	t	value	
	y	$\frac{4}{5}$	1	2	$\frac{1}{5}$	0	0	28	$R_1 \div 5$
	s	$\frac{1}{5}$	0	2	$-\frac{1}{5}$	1	0	32	$R_2 - R_1$
	E	$-\frac{3}{5}$	0	-1	$-\frac{2}{5}$	0	1	4	$R_3 - 2R_1$
	r	-70	0	50	70	0	0	9800	$R_4 + 350R_1$

8 (a)	$SS_1 = 47, SS_2 = 87, T_1, T = 51, T_2, T = 73$ added to diagram	M1 A1 (2)
(b)	$SS_1 \xrightarrow{47}, SS_2 \xleftarrow{49} \xrightarrow{38}, T_1, T \xleftarrow{43} \xrightarrow{8}, T_2, T \xleftarrow{53} \xrightarrow{20}$	M1 A1 (2)
(c)	e.g. $SS_1 A D T, T = 2$ $SS_2 C E T_2, T = 1$ $SS_2 C E D T_2, T = 10$ $SS_2 C E B D T, T = 4$ maximum flow = 113	M1 A4, 3, 2, 1, 0
(d)	e.g. 	(B1) (6) M1 A1 (2)
(e)	max flow - min cut theorem; cut AT, AD, S1B, S2B, BC, CE	(M1) A1 (2)
(f)	Ideal of a <u>directed</u> flow along arcs; from S to T; through a system; practical network	B2, 1, 0 (2) 16

8(e)(m) If all 4 nos. zero then mo
4 arcs added correctly + 4 numbers given (diagram 1 only) condone lack of arrows

A1 c.a.o (diagram 1 only) penalise arrows errors here

(b)m1 4 arcs, 2 numbers and 2 arrows \rightarrow per arc

A1 c.a.o.

(c)m1 2 correct routes + flows found ($\text{flow} > 10$ gets mo) (condone initial f.a. routes only if clearly repeated from new ones.)

A1 all flows + routes found to 17 more.

A3 23 flows+routes to 15 more or flow increased above 17 more.

A2 23 flows + routes to 11 more or

A1 at least 2 flows + routes found to 5 more.

B1 113 c.a.o.

(d)m1 Consistent flow of 101+, complete clear (doesn't need to \checkmark from (c))

A1 correct flow of 113 including arrows

(e)m1 Flow of 113 + cut attempted + max flow - min cut theorem referred to (3 out of 4)

A1 cao

(f) B2 all 4 bits there

B1 2 cut of 4 there.