FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C4

Paper F

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C4 Paper F – Marking Guide

1. $4x + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$ M1 A2

SP: $\frac{dy}{dx} = 0 \quad \therefore 4x + y = 0, \quad y = -4x$ M1 A1

sub. $2x^2 - 4x^2 - 16x^2 + 18 = 0$ M1

$x^2 = 1,\ x = \pm 1 \quad \therefore (-1, 4), (1, -4)$ A2 **(8)**

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2. $x = 2\tan u \Rightarrow \frac{dx}{du} = 2\sec^2 u$ M1

$x = 0 \Rightarrow u = 0,\ x = 2 \Rightarrow u = \frac{\pi}{4}$ B1

$I = \int_0^{\frac{\pi}{4}} \frac{4\tan^2 u}{4\sec^2 u} \times 2\sec^2 u \ du = \int_0^{\frac{\pi}{4}} 2\tan^2 u \ du$ A1

$= \int_0^{\frac{\pi}{4}} (2\sec^2 u - 2) \ du$ M1

$= [2\tan u - 2u]_0^{\frac{\pi}{4}}$ M1 A1

$= (2 - \frac{\pi}{2}) - (0) = \frac{1}{2}(4 - \pi)$ M1 A1 **(8)**

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3. *(a)* $= (\frac{25}{24})^{-\frac{1}{2}} = \sqrt{\frac{24}{25}} = \sqrt{\frac{4}{25}\times 6} = \frac{2}{5}\sqrt{6} \quad [\ k = \frac{2}{5}\ ]$ M1 A1

(b) $= 1 + (-\frac{1}{2})(\frac{1}{2}x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(\frac{1}{2}x)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3\times 2}(\frac{1}{2}x)^3 + \ldots$ M1

$= 1 - \frac{1}{4}x + \frac{3}{32}x^2 - \frac{5}{128}x^3 + \ldots$ A3

(c) $x = \frac{1}{12} \Rightarrow (1 + \frac{1}{2}x)^{-\frac{1}{2}} = (1\frac{1}{24})^{-\frac{1}{2}} = \frac{2}{5}\sqrt{6}$

$x = \frac{1}{12} \Rightarrow (1 + \frac{1}{2}x)^{-\frac{1}{2}} \approx 1 - \frac{1}{4}(\frac{1}{12}) + \frac{3}{32}(\frac{1}{12})^2 - \frac{5}{128}(\frac{1}{12})^3$ M1

$= 0.97979510$

$\therefore \sqrt{6} \approx \frac{5}{2} \times 0.97979510 = 2.44949$ (5dp) M1 A1 **(9)**

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4. *(a)*

$4s = -7 - 3t$ (1)

$7 - 3s = 1$ (2)

$-4 + s = 8 + 2t$ (3) M1

(2) $\Rightarrow s = 2$, sub. (1) $\Rightarrow t = -5$ B1 M1

check (3) $-4 + 2 = 8 - 10$, true $\therefore$ intersect A1

intersect at $(7\mathbf{j} - 4\mathbf{k}) + 2(4\mathbf{i} - 3\mathbf{j} + \mathbf{k}) = (8\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ A1

(b) $= \cos^{-1}\left|\frac{4\times(-3) + (-3)\times 0 + 1\times 2}{\sqrt{16+9+1}\times\sqrt{9+0+4}}\right|$ M1 A1

$= \cos^{-1}\left|\frac{-10}{\sqrt{26}\times\sqrt{13}}\right| = 57.0°$ (1dp) M1 A1 **(9)**

5.	*(a)*	$\frac{dx}{dt} = \frac{1\times(2-t)-t\times(-1)}{(2-t)^2} = \frac{2}{(2-t)^2}$, $\quad \frac{dy}{dt} = -(1+t)^{-2}$	M1 B1	
		$\frac{dy}{dx} = -\frac{1}{(1+t)^2} \div \frac{2}{(2-t)^2} = -\frac{(2-t)^2}{2(1+t)^2} = -\frac{1}{2}\left(\frac{2-t}{1+t}\right)^2$	M1 A1	
	(b)	$t = 1,\ x = 1,\ y = \frac{1}{2}$, grad $= -\frac{1}{8}$	B1	
		grad of normal = 8		
		$\therefore\ y - \frac{1}{2} = 8(x-1) \quad [\ y = 8x - \frac{15}{2}\ ]$	M1 A1	
	(c)	$x(2-t) = t$	M1	
		$2x = t(1+x),\ t = \frac{2x}{1+x}$	A1	
		$y = \frac{1}{1+\frac{2x}{1+x}} = \frac{1+x}{(1+x)+2x} \quad \therefore\ y = \frac{1+x}{1+3x}$	M1 A1	**(11)**
6.	*(a)*	$= \int (\sec^2 x - 1)\ dx$	M1	
		$= \tan x - x + c$	M1 A1	
	(b)	$= \int \frac{\sin x}{\cos x}\ dx$, let $u = \cos x$, $\frac{du}{dx} = -\sin x$	M1	
		$= \int \frac{1}{u} \times (-1)\ du = -\int \frac{1}{u}\ du$	A1	
		$= -\ln\lvert u\rvert + c = \ln\lvert u^{-1}\rvert + c = \ln\lvert \sec x\rvert + c$	M1 A1	
	(c)	volume $= \pi\int_0^{\frac{\pi}{3}} x\tan^2 x\ dx$	M1	
		$u = x,\ u' = 1,\ v' = \tan^2 x,\ v = \tan x - x$	M1	
		$I = x(\tan x - x) - \int (\tan x - x)\ dx$	A1	
		$= x\tan x - x^2 - \ln\lvert \sec x\rvert + \frac{1}{2}x^2 + c$	A1	
		volume $= \pi[x\tan x - \frac{1}{2}x^2 - \ln\lvert \sec x\rvert]_0^{\frac{\pi}{3}}$		
		$= \pi\{(\frac{1}{3}\sqrt{3}\,\pi - \frac{1}{18}\pi^2 - \ln 2) - (0)\} = \frac{1}{18}\pi^2(6\sqrt{3} - \pi) - \pi\ln 2$	M1 A1	**(13)**
7.	*(a)*	$\frac{dV}{dt} = -kV$, $\quad \frac{dV}{dh} = 10\pi h - \pi h^2$	B2	
		$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} \quad \therefore\ -kV = (10\pi h - \pi h^2)\frac{dh}{dt}$	M1	
		$-\frac{1}{3}k\pi h^2(15-h) = \pi h(10-h)\frac{dh}{dt}$		
		$-kh(15-h) = 3(10-h)\frac{dh}{dt} \quad \therefore\ \frac{dh}{dt} = -\frac{kh(15-h)}{3(10-h)}$	M1 A1	
	(b)	$\frac{3(10-h)}{h(15-h)} \equiv \frac{A}{h} + \frac{B}{15-h}$, $\quad 3(10-h) \equiv A(15-h) + Bh$	M1	
		$h = 0 \Rightarrow A = 2,\ h = 15 \Rightarrow B = -1 \quad \therefore\ \frac{3(10-h)}{h(15-h)} \equiv \frac{2}{h} - \frac{1}{15-h}$	A2	
	(c)	$\int \frac{3(10-h)}{h(15-h)}\ dh = \int -k\ dt$, $\quad \int (\frac{2}{h} - \frac{1}{15-h})\ dh = \int -k\ dt$	M1	
		$2\ln\lvert h\rvert + \ln\lvert 15-h\rvert = -kt + c$	M1 A1	
		$t = 0,\ h = 5 \Rightarrow\ 2\ln 5 + \ln 10 = c,\quad c = \ln 250$	M1	
		$2\ln\lvert h\rvert + \ln\lvert 15-h\rvert - \ln 250 = -kt$		
		$\ln\frac{h^2(15-h)}{250} = -kt$, $\quad \frac{h^2(15-h)}{250} = e^{-kt}$, $\quad h^2(15-h) = 250e^{-kt}$	M1 A1	
	(d)	$t = 2,\ h = 4 \Rightarrow\ 176 = 250e^{-2k}$	M1	
		$k = -\frac{1}{2}\ln\frac{176}{250} = 0.175$ (3sf)	M1 A1	**(17)**

Total **(75)**

Performance Record – C4 Paper F

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	differentiation	integration	binomial series	vectors	parametric equations	integration	differential equation, partial fractions	
Marks	8	8	9	9	11	13	17	75
Student								