Mark Scheme (Results)
Summer 2007

GCE

GCE Mathematics

Core Mathematics C4 (6666)

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Question Number	Scheme		Marks
	** represents a constant		
1. (a)	$\mathrm{f}(x) = (3+2x)^{-3} = \underline{(3)^{-3}}\left(1+\frac{2x}{3}\right)^{-3} = \underline{\frac{1}{27}}\left(1+\frac{2x}{3}\right)^{-3}$	Takes 3 outside the bracket to give any of $(3)^{-3}$ or $\frac{1}{27}$. See note below.	B1
	$= \frac{1}{27}\underline{\left\{1+(-3)(**x);+\frac{(-3)(-4)}{2!}(**x)^2+\frac{(-3)(-4)(-5)}{3!}(**x)^3+\ldots\right\}}$ with $** \neq 1$	Expands $(1+**x)^{-3}$ to give a simplified or an un-simplified $1+(-3)(**x)$; A correct simplified or an un-simplified $\{\underline{\ldots\ldots\ldots}\}$ expansion with candidate's followed thro' $(**x)$	M1; A1 $\sqrt{}$
	$= \frac{1}{27}\underline{\left\{1+(-3)(\tfrac{2x}{3})+\frac{(-3)(-4)}{2!}(\tfrac{2x}{3})^2+\frac{(-3)(-4)(-5)}{3!}(\tfrac{2x}{3})^3+\ldots\right\}}$		
	$= \frac{1}{27}\left\{1-2x+\frac{8x^2}{3}-\frac{80}{27}x^3+\ldots\right\}$		
	$= \frac{1}{27}-\frac{2x}{27};+\frac{8x^2}{81}-\frac{80x^3}{729}+\ldots$	Anything that cancels to $\frac{1}{27}-\frac{2x}{27}$; Simplified $\frac{8x^2}{81}-\frac{80x^3}{729}$	A1; A1 [5]
			5 marks

Note: You would award: B1M1A0 for

$$= \frac{1}{27}\underline{\left\{1+(-3)(\tfrac{2x}{3})+\frac{(-3)(-4)}{2!}(2x)^2+\frac{(-3)(-4)(-5)}{3!}(2x)^3+\ldots\right\}}$$

because $**$ is not consistent.

Special Case: If you see the constant $\frac{1}{27}$ in a candidate's final binomial expression, then you can award B1

Question Number	Scheme		Marks
Aliter 1. Way 2	$f(x) = (3 + 2x)^{-3}$		
		$\frac{1}{27}$ or $(3)^{-3}$ (See note ↓)	B1
	$= \left\{ \begin{array}{l} (3)^{-3} + (-3)(3)^{-4}(**x); + \frac{(-3)(-4)}{2!}(3)^{-5}(**x)^2 \\ + \frac{(-3)(-4)(-5)}{3!}(3)^{-6}(**x)^3 + \ldots \end{array} \right\}$	Expands $(3+2x)^{-3}$ to give an un-simplified or simplified $(3)^{-3} + (-3)(3)^{-4}(**x)$;	M1
	with $** \neq 1$	A correct un-simplified or simplified $\{\ldots\ldots\}$ expansion with candidate's followed thro' $(**x)$	A1 $\sqrt{}$
	$= \left\{ \begin{array}{l} (3)^{-3} + (-3)(3)^{-4}(2x); + \frac{(-3)(-4)}{2!}(3)^{-5}(2x)^2 \\ + \frac{(-3)(-4)(-5)}{3!}(3)^{-6}(2x)^3 + \ldots \end{array} \right\}$		
	$= \left\{ \begin{array}{l} \frac{1}{27} + (-3)(\frac{1}{81})(2x); + (6)(\frac{1}{243})(4x^2) \\ + (-10)(\frac{1}{729})(8x^3) + \ldots \end{array} \right\}$		
	$= \frac{1}{27} - \frac{2x}{27}; + \frac{8x^2}{81} - \frac{80x^3}{729} + \ldots$	Anything that cancels to $\frac{1}{27} - \frac{2x}{27}$;	A1;
		Simplified $\frac{8x^2}{81} - \frac{80x^3}{729}$	A1
			[5]
			5 marks

Attempts using Maclaurin expansions need to be escalated up to your team leader.

If you feel the mark scheme does not apply fairly to a candidate please escalate the response up to your team leader.

Special Case: If you see the constant $\frac{1}{27}$ in a candidate's final binomial expression, then you can award B1

Question Number	Scheme		Marks
2.	$\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx$, with substitution $u = 2^x$		
	$\frac{du}{dx} = 2^x.\ln 2 \Rightarrow \frac{dx}{du} = \frac{1}{2^x.\ln 2}$	$\frac{du}{dx} = 2^x.\ln 2$ or $\frac{du}{dx} = u.\ln 2$ or $\left(\frac{1}{u}\right)\frac{du}{dx} = \ln 2$	B1
	$\int \frac{2^x}{(2^x+1)^2}\,dx = \left(\frac{1}{\ln 2}\right)\int \frac{1}{(u+1)^2}\,du$	$k\int \frac{1}{(u+1)^2}\,du$ where k is constant	M1*
	$= \left(\frac{1}{\ln 2}\right)\left(\frac{-1}{(u+1)}\right) + c$	$(u+1)^{-2} \to a(u+1)^{-1}$ $(u+1)^{-2} \to -1.(u+1)^{-1}$	M1 A1
	change limits: when $x = 0$ & $x = 1$ then $u = 1$ & $u = 2$		
	$\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx = \frac{1}{\ln 2}\left[\frac{-1}{(u+1)}\right]_1^2$		
	$= \frac{1}{\ln 2}\left[\left(-\frac{1}{3}\right) - \left(-\frac{1}{2}\right)\right]$	Correct use of limits $u = 1$ and $u = 2$	depM1*
	$= \frac{1}{6\ln 2}$	$\frac{1}{6\ln 2}$ or $\frac{1}{\ln 4} - \frac{1}{\ln 8}$ or $\frac{1}{2\ln 2} - \frac{1}{3\ln 2}$ Exact value only!	A1 aef [6]
	Alternatively candidate can revert back to x …		
	$\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx = \frac{1}{\ln 2}\left[\frac{-1}{(2^x+1)}\right]_0^1$		
	$= \frac{1}{\ln 2}\left[\left(-\frac{1}{3}\right) - \left(-\frac{1}{2}\right)\right]$	Correct use of limits $x = 0$ and $x = 1$	depM1*
	$= \underline{\frac{1}{6\ln 2}}$	$\frac{1}{6\ln 2}$ or $\frac{1}{\ln 4} - \frac{1}{\ln 8}$ or $\frac{1}{2\ln 2} - \frac{1}{3\ln 2}$ Exact value only!	A1 aef
			6 marks

If you see this **integration** applied anywhere in a candidate's working then you can award M1, A1

There are other acceptable answers for A1, eg: $\frac{1}{2\ln 8}$ or $\frac{1}{\ln 64}$

NB: Use your calculator to check eg. 0.240449…

Question Number	Scheme		Marks
3. (a)	$\left\{\begin{array}{ll} u = x & \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = \cos 2x & \Rightarrow v = \frac{1}{2}\sin 2x \end{array}\right\}$	*(see note below)*	
	$\text{Int} = \int x\cos 2x \, dx = \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x.1 \, dx$	Use of 'integration by parts' formula in the correct direction. Correct expression.	M1 A1
	$= \frac{1}{2}x\sin 2x - \frac{1}{2}\left(-\frac{1}{2}\cos 2x\right) + c$	$\sin 2x \to -\frac{1}{2}\cos 2x$ or $\sin kx \to -\frac{1}{k}\cos kx$ with $k \neq 1, k > 0$	dM1
	$= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$	Correct expression with +*c*	A1 [4]
(b)	$\int x\cos^2 x \, dx = \int x\left(\frac{\cos 2x + 1}{2}\right) dx$	Substitutes correctly for $\cos^2 x$ in the given integral	M1
	$= \frac{1}{2}\int x\cos 2x \, dx + \frac{1}{2}\int x \, dx$		
	$= \underline{\frac{1}{2}\left(\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right)}; + \frac{1}{2}\int x \, dx$	$\frac{1}{2}$(their answer to (a)); or underlined expression	A1; $\sqrt{}$
	$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2 \ (+c)$	Completely correct expression with/without +*c*	A1 [3]
			7 marks

Notes:

(b)	$\text{Int} = \int x\cos 2x \, dx = \frac{1}{2}x\sin 2x \pm \int \frac{1}{2}\sin 2x.1 \, dx$	This is acceptable for M1	M1
	$\left\{\begin{array}{ll} u = x & \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = \cos 2x & \Rightarrow v = \lambda\sin 2x \end{array}\right\}$ $\text{Int} = \int x\cos 2x \, dx = \lambda x\sin 2x \pm \int \lambda\sin 2x.1 \, dx$	This is also acceptable for M1	M1

Question Number	Scheme		Marks
Aliter 3. (b) Way 2	$\int x\cos^2 x\,dx = \int x\left(\frac{\cos 2x+1}{2}\right)dx$	Substitutes <u>correctly</u> for $\cos^2 x$ in the given integral …	M1
	$\left\{\begin{array}{ll} u = x & \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = \frac{1}{2}\cos 2x + \frac{1}{2} & \Rightarrow v = \frac{1}{4}\sin 2x + \frac{1}{2}x \end{array}\right\}$	… or $u = x$ and $\frac{dv}{dx} = \frac{1}{2}\cos 2x + \frac{1}{2}$	
	$= \frac{1}{4}x\sin 2x + \frac{1}{2}x^2 - \int\left(\frac{1}{4}\sin 2x + \frac{1}{2}x\right)dx$		
	$= \underline{\frac{1}{4}x\sin 2x} + \frac{1}{2}x^2 + \underline{\frac{1}{8}\cos 2x} - \frac{1}{4}x^2 + c$	$\frac{1}{2}$(their answer to (a)); or <u>underlined expression</u>	A1 $\sqrt{}$
	$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2\ (+c)$	Completely correct expression with/without $+c$	A1
			[3]
Aliter (b) Way 3	$\int x\cos 2x\,dx = \int x\left(2\cos^2 x - 1\right)dx$	Substitutes <u>correctly</u> for $\cos 2x$ in $\int x\cos 2x\,dx$	M1
	$\Rightarrow 2\int x\cos^2 x\,dx - \int x\,dx = \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$		
	$\Rightarrow \int x\cos^2 x\,dx = \underline{\frac{1}{2}\left(\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right)}; + \frac{1}{2}\int x\,dx$	$\frac{1}{2}$(their answer to (a)); or <u>underlined expression</u>	A1; $\sqrt{}$
	$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2\ (+c)$	Completely correct expression with/without $+c$	A1
			[3]
			7 marks

Question Number	Scheme		Marks
4. (a) Way 1	A method of long division gives,		
	$\dfrac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv 2 + \dfrac{4}{(2x+1)(2x-1)}$	$A = 2$	B1
	$\dfrac{4}{(2x+1)(2x-1)} \equiv \dfrac{B}{(2x+1)} + \dfrac{C}{(2x-1)}$		
	$4 \equiv B(2x-1) + C(2x+1)$ or their remainder, $Dx + E \equiv B(2x-1) + C(2x+1)$	Forming any one of these two identities. Can be implied.	M1
	Let $x = -\frac{1}{2}$, $4 = -2B \Rightarrow B = -2$		
	Let $x = \frac{1}{2}$, $4 = 2C \Rightarrow C = 2$	See note below either one of $B = -2$ or $C = 2$ both B and C correct	A1 A1 [4]
Aliter 4. (a) Way 2	$\dfrac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv A + \dfrac{B}{(2x+1)} + \dfrac{C}{(2x-1)}$		
	See below for the award of B1	*decide to award B1 here!! …* *… for* $A = 2$	B1
	$2(4x^2+1) \equiv A(2x+1)(2x-1) + B(2x-1) + C(2x+1)$	Forming this identity. Can be implied.	M1
	Equate x^2, $8 = 4A \Rightarrow A = 2$		
	Let $x = -\frac{1}{2}$, $4 = -2B \Rightarrow B = -2$		
	Let $x = \frac{1}{2}$, $4 = 2C \Rightarrow C = 2$	See note below either one of $B = -2$ or $C = 2$ both B and C correct	A1 A1 [4]

If a candidate states one of either B or C correctly then the method mark M1 can be implied.

Question Number	Scheme		Marks
4. (b)	$\int \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,dx = \int 2 - \frac{2}{(2x+1)} + \frac{2}{(2x-1)}\,dx$		
	$= 2x - \frac{2}{2}\ln(2x+1) + \frac{2}{2}\ln(2x-1)\ (+c)$	Either $p\ln(2x+1)$ or $q\ln(2x-1)$ or either $p\ln 2x+1$ or $q\ln 2x-1$	M1 *
		$A \to Ax$	B1 $\sqrt{}$
		$-\frac{2}{2}\ln(2x+1) + \frac{2}{2}\ln(2x-1)$ or $-\ln(2x+1) + \ln(2x-1)$ See note below.	A1 cso & aef
	$\int_1^2 \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,dx = \left[2x - \ln(2x+1) + \ln(2x-1)\right]_1^2$		
	$= (4 - \ln 5 + \ln 3) - (2 - \ln 3 + \ln 1)$	Substitutes limits of 2 and 1 and subtracts the correct way round. (Invisible brackets okay.)	depM1 *
	$= 2 + \ln 3 + \ln 3 - \ln 5$		
	$= 2 + \ln\left(\frac{3(3)}{5}\right)$	Use of correct product (or power) and/or quotient laws for logarithms to obtain a single logarithmic term for *their numerical* expression.	M1
	$= 2 + \ln\left(\frac{9}{5}\right)$	$2 + \ln\left(\frac{9}{5}\right)$ Or $2 - \ln\left(\frac{5}{9}\right)$ and k stated as $\frac{9}{5}$.	A1 [6]
			10 marks

Some candidates may find rational values for B and C. They may combine the denominator of their B or C with (2x +1) or (2x – 1). Hence: Either $\frac{a}{b(2x-1)} \to k\ln(b(2x-1))$ or $\frac{a}{b(2x+1)} \to k\ln(b(2x+1))$ is okay for M1.

Candidates are not allowed to fluke $-\ln(2x+1) + \ln(2x-1)$ for A1. Hence **cso**. If they do fluke this, however, they can gain the final A1 mark for this part of the question.

To award this M1 mark, the candidate must use the appropriate law(s) of logarithms for their ln terms to give a **one single** logarithmic term. Any error ***in applying the laws of logarithms*** would then earn M0.

Note: This is not a dependent method mark.

Question Number	Scheme		Marks
5. (a)	If l_1 and l_2 intersect then:		
	$\begin{pmatrix}1\\0\\-1\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}1\\3\\6\end{pmatrix} + \mu\begin{pmatrix}2\\1\\-1\end{pmatrix}$		
	Any two of **i**: $1+\lambda = 1+2\mu$ (1); **j**: $\lambda = 3+\mu$ (2); **k**: $-1 = 6-\mu$ (3)	Writes down any two of these equations correctly.	M1
	(1) & (2) yields $\lambda = 6,\ \mu = 3$ (1) & (3) yields $\lambda = 14,\ \mu = 7$ (2) & (3) yields $\lambda = 10,\ \mu = 7$	Solves two of the above equations to find … either one of λ or μ correct both λ and μ correct	A1 A1
	Either checking eqn (3), $-1 \neq 3$ checking eqn (2), $14 \neq 10$ checking eqn (1), $11 \neq 15$	Complete method of putting their values of λ and μ into a third equation to show a contradiction.	B1 $\sqrt{}$
	or for example: checking eqn (3), LHS = -1, RHS = 3 $\Rightarrow$ Lines l_1 and l_2 do not intersect	this type of explanation is also allowed for B1 $\sqrt{}$.	
			[4]
Aliter 5. (a) Way 2	**k**: $-1 = 6 - \mu \Rightarrow \mu = 7$		
	i: $1+\lambda = 1+2\mu \Rightarrow 1+\lambda = 1+2(7)$ **j**: $\lambda = 3+\mu \Rightarrow \lambda = 3+(7)$	Uses the k component to find μ and substitutes their value of μ into either one of the i or j component.	M1
	i: $\lambda = 14$ **j**: $\lambda = 10$	either one of the λ's correct both of the λ's correct	A1 A1
	Either: These equations are then inconsistent Or: $14 \neq 10$ Or: Lines l_1 and l_2 do not intersect	Complete method giving rise to any one of these three explanations.	B1 $\sqrt{}$
			[4]

Question Number	Scheme		Marks
Aliter 5. (a) Way 3	If l_1 and l_2 intersect then:		
	$\begin{pmatrix}1\\0\\-1\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}1\\3\\6\end{pmatrix} + \mu\begin{pmatrix}2\\1\\-1\end{pmatrix}$		
	Any two of $\mathbf{i}: \quad 1+\lambda = 1+2\mu \quad (1)$ $\mathbf{j}: \quad \lambda = 3+\mu \quad (2)$ $\mathbf{k}: \quad -1 = 6-\mu \quad (3)$	Writes down any two of these equations	M1
	(1) & (2) yields $\mu = 3$	either one of the μ's correct	A1
	(3) yields $\mu = 7$	both of the μ's correct	A1
	Either: These equations are then inconsistent Or: $3 \neq 7$ Or: Lines l_1 and l_2 do not intersect	Complete method giving rise to any one of these three explanations.	B1 $\sqrt{}$
			[4]
Aliter 5. (a) Way 4	Any two of $\mathbf{i}: \quad 1+\lambda = 1+2\mu \quad (1)$ $\mathbf{j}: \quad \lambda = 3+\mu \quad (2)$ $\mathbf{k}: \quad -1 = 6-\mu \quad (3)$	Writes down any two of these equations	M1
	(1) & (2) yields $\mu = 3$	$\mu = 3$	A1
	(3) RHS $= 6-3 = 3$	RHS of (3) = 3	A1
	(3) yields $-1 \neq 3$	Complete method giving rise to this explanation.	B1 $\sqrt{}$
			[4]

Question Number	Scheme		Marks
5. (b)	$\lambda = 1 \Rightarrow \overrightarrow{OA} = \begin{pmatrix}2\\1\\-1\end{pmatrix}$ & $\mu = 2 \Rightarrow \overrightarrow{OB} = \begin{pmatrix}5\\5\\4\end{pmatrix}$	Only one of either $\overrightarrow{OA} = \begin{pmatrix}2\\1\\-1\end{pmatrix}$ or $\overrightarrow{OB} = \begin{pmatrix}5\\5\\4\end{pmatrix}$ or $A(2,1,-1)$ or $B(5,5,4)$. (can be implied)	B1
	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix}5\\5\\4\end{pmatrix} - \begin{pmatrix}2\\1\\-1\end{pmatrix} = \begin{pmatrix}3\\4\\5\end{pmatrix}$ or $\overrightarrow{BA} = \begin{pmatrix}-3\\-4\\-5\end{pmatrix}$	Finding the difference between their $\overrightarrow{OB}$ and $\overrightarrow{OA}$. (can be implied)	M1 $\sqrt{}$
		Applying the dot product formula between "allowable" vectors. See notes below.	M1
	$\overrightarrow{AB} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$, $\mathbf{d}_1 = \mathbf{i} + \mathbf{j} + 0\mathbf{k}$ & θ is angle		
	$\cos\theta = \dfrac{\overrightarrow{AB} \bullet \mathbf{d}_1}{\lvert\overrightarrow{AB}\rvert . \lvert\mathbf{d}_1\rvert} = \pm\left(\dfrac{3+4+0}{\sqrt{50}.\sqrt{2}}\right)$	Applies dot product formula between $\mathbf{d}_1$ and their $\pm\overrightarrow{AB}$.	M1 $\sqrt{}$
		Correct expression.	A1
	$\cos\theta = \frac{7}{10}$	$\frac{7}{10}$ or 0.7 or $\frac{7}{\sqrt{100}}$ but not $\frac{7}{\sqrt{50}\sqrt{2}}$	A1 cao
			[6]
			10 marks

Candidates can score this mark if there is a complete method for finding the dot product between their vectors in the following cases:

Case 1: their ft $\pm\overrightarrow{AB} = \pm(3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k})$ and $\mathbf{d}_1 = \mathbf{i} + \mathbf{j} + 0\mathbf{k}$
$\Rightarrow \cos\theta = \pm\left(\dfrac{3+4+0}{\sqrt{50}.\sqrt{2}}\right)$

Case 2: $\mathbf{d}_1 = \mathbf{i} + \mathbf{j} + 0\mathbf{k}$ and $\mathbf{d}_2 = 2\mathbf{i} + \mathbf{j} - 1\mathbf{k}$
$\Rightarrow \cos\theta = \dfrac{2+1+0}{\sqrt{2}.\sqrt{6}}$

Case 3: $\mathbf{d}_1 = \mathbf{i} + \mathbf{j} + 0\mathbf{k}$ and $\mathbf{d}_2 = 2(2\mathbf{i} + \mathbf{j} - 1\mathbf{k})$
$\Rightarrow \cos\theta = \dfrac{4+2+0}{\sqrt{2}.\sqrt{24}}$

Case 4: their ft $\pm\overrightarrow{AB} = \pm(3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k})$ and $\mathbf{d}_2 = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$
$\Rightarrow \cos\theta = \pm\left(\dfrac{6+4-5}{\sqrt{50}.\sqrt{6}}\right)$

Case 5: their ft $\overrightarrow{OA} = 2\mathbf{i} + 1\mathbf{j} - 1\mathbf{k}$ and their ft $\overrightarrow{OB} = 5\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$
$\Rightarrow \cos\theta = \pm\left(\dfrac{10+5-4}{\sqrt{6}.\sqrt{66}}\right)$

Note: If candidate use cases 2, 3, 4 and 5 they cannot gain the final three marks for this part.
Note: Candidate can only gain some/all of the final three marks if they use case 1.

Examples of awarding of marks M1M1A1 in 5.(b)

Example	Marks
$\sqrt{50}.\sqrt{2}\cos\theta = \pm(3+4+0)$	M1M1A1 (Case 1)
$\sqrt{2}.\sqrt{6}\cos\theta = 3$	M1M0A0 (Case 2)
$\sqrt{2}.\sqrt{24}\cos\theta = 4+2$	M1M0A0 (Case 3)

Question Number	Scheme		Marks
6. (a)	$x = \tan^2 t$, $\quad y = \sin t$		
	$\frac{dx}{dt} = 2(\tan t)\sec^2 t$, $\frac{dy}{dt} = \cos t$	Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	B1
	$\therefore \frac{dy}{dx} = \frac{\cos t}{2\tan t \sec^2 t} \quad \left(= \frac{\cos^4 t}{2\sin t} \right)$	$\frac{\pm\cos t}{\text{their } \frac{dx}{dt}}$	M1
		$\frac{+\cos t}{\text{their } \frac{dx}{dt}}$	A1 $\sqrt{}$
			[3]
(b)	When $t = \frac{\pi}{4}$, $\quad x = 1$, $y = \frac{1}{\sqrt{2}}$ (need values)	The point $\underline{\left(1, \frac{1}{\sqrt{2}}\right)}$ or $\underline{(1, \text{awrt } 0.71)}$ These coordinates can be implied. ($y = \sin\left(\frac{\pi}{4}\right)$ is not sufficient for B1)	B1, B1
	When $t = \frac{\pi}{4}$, m(T) $= \frac{dy}{dx} = \frac{\cos\frac{\pi}{4}}{2\tan\frac{\pi}{4}\sec^2\frac{\pi}{4}}$		
	$= \underline{\frac{\frac{1}{\sqrt{2}}}{2.(1)\left(\frac{1}{\frac{1}{\sqrt{2}}}\right)^2}} = \underline{\frac{\frac{1}{\sqrt{2}}}{2.(1)\left(\frac{1}{\frac{1}{2}}\right)}} = \underline{\frac{\frac{1}{\sqrt{2}}}{2.(1)(2)}} = \underline{\frac{1}{4\sqrt{2}}} = \underline{\frac{\sqrt{2}}{8}}$	any of the five underlined expressions or awrt 0.18	B1 aef
	T: $y - \frac{1}{\sqrt{2}} = \frac{1}{4\sqrt{2}}(x - 1)$	Finding an equation of a tangent with *their point* and *their tangent gradient* or finds *c* by using $y = (\underline{\text{their gradient}})x + "\underline{c}"$.	M1 $\sqrt{}$ aef
	T: $\underline{y = \frac{1}{4\sqrt{2}}x + \frac{3}{4\sqrt{2}}}$ or $\underline{y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}}$	Correct simplified EXACT equation of <u>tangent</u>	<u>A1</u> aef cso
	or $\frac{1}{\sqrt{2}} = \frac{1}{4\sqrt{2}}(1) + c \Rightarrow c = \frac{1}{\sqrt{2}} - \frac{1}{4\sqrt{2}} = \frac{3}{4\sqrt{2}}$		
	Hence T: $\underline{y = \frac{1}{4\sqrt{2}}x + \frac{3}{4\sqrt{2}}}$ or $\underline{y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}}$		
			[5]

Note: The x and y coordinates must be the right way round.

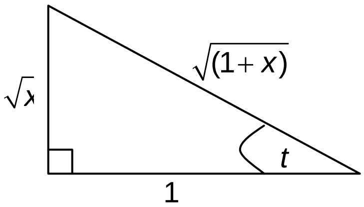
A candidate who incorrectly differentiates $\tan^2 t$ to give $\frac{dx}{dt} = 2\sec^2 t$ or $\frac{dx}{dt} = \sec^4 t$ is then able to fluke the correct answer in part (b). Such candidates can potentially get: (a) B0M1A1 $\sqrt{}$ (b) B1B1B1M1A0 **cso**.
Note: cso means "correct solution only".
Note: part (a) not fully correct implies candidate can achieve a maximum of 4 out of 5 marks in part (b).

Question Number	Scheme		Marks
6. (c) Way 1	$x = \tan^2 t = \dfrac{\sin^2 t}{\cos^2 t}$ $\quad y = \sin t$		
	$x = \dfrac{\sin^2 t}{1 - \sin^2 t}$	Uses $\cos^2 t = 1 - \sin^2 t$	M1
	$x = \dfrac{y^2}{1 - y^2}$	Eliminates '*t*' *to* write an equation involving *x* and *y*.	M1
	$x(1 - y^2) = y^2 \Rightarrow x - xy^2 = y^2$		
	$x = y^2 + xy^2 \Rightarrow x = y^2(1 + x)$	Rearranging and factorising with an attempt to make y^2 the subject.	ddM1
	$y^2 = \dfrac{x}{1 + x}$	$\dfrac{x}{1 + x}$	A1
			[4]
Aliter 6. (c) Way 2	$1 + \cot^2 t = co\sec^2 t$	Uses $1 + \cot^2 t = co\sec^2 t$	M1
	$= \dfrac{1}{\sin^2 t}$	Uses $\cos ec^2 t = \dfrac{1}{\sin^2 t}$	M1 implied
	Hence, $\quad 1 + \dfrac{1}{x} = \dfrac{1}{y^2}$	Eliminates '*t*' *to* write an equation involving *x* and *y*.	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1 + x)}$ or $\dfrac{x}{1 + x}$	$1 - \dfrac{1}{(1 + x)}$ or $\dfrac{x}{1 + x}$	A1
			[4]

$\dfrac{1}{1 + \frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

Question Number	Scheme		Marks
Aliter 6. (c) Way 3	$x = \tan^2 t \qquad y = \sin t$		
	$1 + \tan^2 t = \sec^2 t$	Uses $1 + \tan^2 t = \sec^2 t$	M1
	$= \dfrac{1}{\cos^2 t}$	Uses $\sec^2 t = \dfrac{1}{\cos^2 t}$	M1
	$= \dfrac{1}{1 - \sin^2 t}$		
	Hence, $1 + x = \dfrac{1}{1 - y^2}$	Eliminates '*t*' *to* write an equation involving *x* and *y*.	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	$1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	A1
			[4]
Aliter 6. (c) Way 4	$y^2 = \sin^2 t = 1 - \cos^2 t$	Uses $\sin^2 t = 1 - \cos^2 t$	M1
	$= 1 - \dfrac{1}{\sec^2 t}$	Uses $\cos^2 t = \dfrac{1}{\sec^2 t}$	M1
	$= 1 - \dfrac{1}{(1 + \tan^2 t)}$	then uses $\sec^2 t = 1 + \tan^2 t$	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	$1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	A1
			[4]

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

Question Number	Scheme		Marks
Aliter 6. (c) Way 5	$x = \tan^2 t \qquad y = \sin t$		
	$x = \tan^2 t \;\Rightarrow\; \tan t = \sqrt{x}$		
		Draws a right-angled triangle and places both $\sqrt{x}$ and 1 on the triangle	M1
		Uses Pythagoras to deduce the hypotenuse	M1
	Hence, $y = \sin t = \dfrac{\sqrt{x}}{\sqrt{1+x}}$	Eliminates 't' *to* write an equation involving x and y.	ddM1
	Hence, $y^2 = \dfrac{x}{1+x}$	$\dfrac{x}{1+x}$	A1
			[4]
			12 marks

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

There are so many ways that a candidate can proceed with part (c). If a candidate produces a correct solution then please award all four marks. If they use a method commensurate with the five ways as detailed on the mark scheme then award the marks appropriately. If you are unsure of how to apply the scheme please escalate your response up to your team leader.

Question Number	Scheme		Marks
7. (a)	x: 0, $\frac{\pi}{16}$, $\frac{\pi}{8}$, $\frac{3\pi}{16}$, $\frac{\pi}{4}$ y: 0, $0.445995927\ldots$, $0.643594252\ldots$, $0.817421946\ldots$, 1		
	Enter marks into ePEN in the correct order.	0.446 or awrt 0.44600 awrt 0.64359 awrt 0.81742	B1 B1 B1 [3]
(b) Way 1	[0 can be implied] $\text{Area} \approx \frac{1}{2} \times \frac{\pi}{16}\ ; \times \underline{\left\{0 + 2\left(0.44600 + 0.64359 + 0.81742\right) + 1\right\}}$	Outside brackets $\frac{1}{2} \times \frac{\pi}{16}$ or $\frac{\pi}{32}$ For structure of trapezium rule $\{\ldots\ldots\ldots\ldots\}$; Correct expression inside brackets which all must be multiplied by $\frac{h}{2}$.	B1 M1 √ A1 √
	$= \frac{\pi}{32} \times 4.81402\ldots = 0.472615308\ldots = \underline{0.4726}$ (4dp)	for seeing $\underline{0.4726}$	A1 cao [4]
Aliter (b) Way 2	$\text{Area} \approx \frac{\pi}{16} \times \left\{\frac{0+0.44600}{2} + \frac{0.44600+0.64359}{2} + \frac{0.64359+0.81742}{2} + \frac{0.81742+1}{2}\right\}$ which is equivalent to: $\text{Area} \approx \frac{1}{2} \times \frac{\pi}{16}\ ; \times \underline{\left\{0 + 2\left(0.44600 + 0.64359 + 0.81742\right) + 1\right\}}$	$\frac{\pi}{16}$ and a divisor of 2 on all terms inside brackets. One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2. Correct expression inside brackets if $\frac{1}{2}$ was to be factorised out.	B1 M1 √ A1 √
	$= \frac{\pi}{16} \times 2.40701\ldots = 0.472615308\ldots = \underline{0.4726}$	$\underline{0.4726}$	A1 cao [4]

$Area = \frac{1}{2} \times \frac{\pi}{20} \times \{0 + 2(0.44600 + 0.64359 + 0.81742) + 1\} = 0.3781$, gains B0M1A1A0

In (a) for $x = \frac{\pi}{16}$ writing 0.4459959… then 0.45600 gains B1 for awrt 0.44600 even though 0.45600 is incorrect.

In (b) you can follow though a candidate's values from part (a) to award M1 ft, A1 ft

Question Number	Scheme	Marks

7. (c)	Volume $= (\pi)\int_0^{\frac{\pi}{4}} \left(\sqrt{\tan x}\right)^2 dx = (\pi)\int_0^{\frac{\pi}{4}} \tan x\, dx$	$\int \left(\sqrt{\tan x}\right)^2 dx$ or $\int \tan x\, dx$ Can be implied. Ignore limits and (π)	M1
	$= (\pi)\left[\ln \sec x\right]_0^{\frac{\pi}{4}}$ or $= (\pi)\left[-\ln \cos x\right]_0^{\frac{\pi}{4}}$	$\tan x \to \ln \sec x$ or $\tan x \to -\ln \cos x$	A1
	$= (\pi)\left[\left(\ln \sec \frac{\pi}{4}\right) - \left(\ln \sec 0\right)\right]$ or $= (\pi)\left[\left(-\ln \cos \frac{\pi}{4}\right) - \left(\ln \cos 0\right)\right]$	The correct use of limits on a function other than tan x; ie $x = \frac{\pi}{4}$ 'minus' $x = 0$. $\ln(\sec 0) = 0$ may be implied. Ignore (π)	dM1
	$= \pi\left[\ln\left(\frac{1}{\frac{1}{\sqrt{2}}}\right) - \ln\left(\frac{1}{1}\right)\right] = \pi\left[\ln\sqrt{2} - \ln 1\right]$ or $= \pi\left[-\ln\left(\frac{1}{\sqrt{2}}\right) - \ln(1)\right]$		
	$= \pi \ln\sqrt{2}$ or $\pi \ln \frac{2}{\sqrt{2}}$ or $\frac{1}{2}\pi \ln 2$ or $-\pi \ln\left(\frac{1}{\sqrt{2}}\right)$ or $\frac{\pi}{2}\ln\left(\frac{1}{2}\right)$	$\pi \ln\sqrt{2}$ or $\pi \ln \frac{2}{\sqrt{2}}$ or $\frac{1}{2}\pi \ln 2$ or $-\pi \ln\left(\frac{1}{\sqrt{2}}\right)$ or $\frac{\pi}{2}\ln\left(\frac{1}{2}\right)$ must be exact.	A1 aef [4]
			11 marks

If a candidate gives the correct exact answer and then writes 1.088779…, then such a candidate can be awarded A1 (aef). The subsequent working would then be ignored. (isw)

Beware: In part (c) the factor of π is not needed for the first three marks.

Beware: In part (b) a candidate can also add up individual trapezia in this way:

$$\text{Area} \approx \tfrac{1}{2}.\tfrac{\pi}{16}\left(0 + 0.44600\right) + \tfrac{1}{2}.\tfrac{\pi}{16}\left(0.44600 + 0.64359\right) + \tfrac{1}{2}.\tfrac{\pi}{16}\left(0.64359 + 0.81742\right) + \tfrac{1}{2}.\tfrac{\pi}{16}\left(0.81742 + 1\right)$$

Question Number	Scheme		Marks
8. (a)	$\frac{dP}{dt} = kP$ and $t = 0,\ P = P_0$ (1)		
	$\int \frac{dP}{P} = \int k\ dt$	Separates the variables with $\int \frac{dP}{P}$ and $\int k\,dt$ on either side with integral signs not necessary.	M1
	$\ln P = kt;(+c)$	Must see $\ln P$ and kt; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \ \Rightarrow\ \ln P_0 = c$ $\left(\text{or } P = Ae^{kt} \ \Rightarrow\ P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\ln P = kt + \ln P_0 \ \Rightarrow e^{\ln P} = e^{kt + \ln P_0} = e^{kt}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{kt}}$	$\underline{P = P_0 e^{kt}}$	A1
			[4]
(b)	$P = 2P_0$ & $k = 2.5 \ \Rightarrow\ \underline{2P_0 = P_0 e^{2.5t}}$	Substitutes $P = 2P_0$ into an expression involving P	M1
	$e^{2.5t} = 2 \ \Rightarrow\ \underline{\ln e^{2.5t} = \ln 2}$ or $\underline{2.5t = \ln 2}$ …or $e^{kt} = 2 \ \Rightarrow\ \underline{\ln e^{kt} = \ln 2}$ or $\underline{kt = \ln 2}$	Eliminates P_0 and takes ln of both sides	M1
	$\Rightarrow\ t = \frac{1}{2.5}\ln 2 = 0.277258872...$ days		
	$t = 0.277258872... \times 24 \times 60 = 399.252776...$ minutes		
	$t = \underline{399\text{min}}$ or $t = \underline{6\text{ hr } 39\text{ mins}}$ (to nearest minute)	awrt $t = \underline{399}$ or $\underline{6\text{ hr } 39\text{ mins}}$	A1
			[3]

$\underline{P = P_0 e^{kt}}$ written down without the first M1 mark given scores all four marks in part (a).

Question Number	Scheme		Marks
8. (c)	$\frac{dP}{dt} = \lambda P \cos \lambda t$ and $t = 0,\ P = P_0$ (1)		
	$\int \frac{dP}{P} = \int \lambda \cos \lambda t \ dt$	Separates the variables with $\int \frac{dP}{P}$ and $\int \lambda \cos \lambda t \, dt$ on either side with integral signs not necessary.	M1
	$\ln P = \sin \lambda t; (+c)$	Must see $\ln P$ and $\sin \lambda t$; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \ln P_0 = c$ $\left(\text{or } P = Ae^{\sin \lambda t} \Rightarrow P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\ln P = \sin \lambda t + \ln P_0 \Rightarrow e^{\ln P} = e^{\sin \lambda t + \ln P_0} = e^{\sin \lambda t}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{\sin \lambda t}}$	$\underline{P = P_0 e^{\sin \lambda t}}$	A1
			[4]
(d)	$P = 2P_0$ & $\lambda = 2.5 \Rightarrow 2P_0 = P_0 e^{\sin 2.5t}$		
	$e^{\sin 2.5t} = 2 \Rightarrow \underline{\sin 2.5t = \ln 2}$ …or … $e^{\lambda t} = 2 \Rightarrow \underline{\sin \lambda t = \ln 2}$	Eliminates P_0 and makes $\sin \lambda t$ or $\sin 2.5t$ the subject by taking ln's	M1
	$\underline{t = \frac{1}{2.5} \sin^{-1}(\ln 2)}$	Then rearranges to make *t* the subject. (must use $\sin^{-1}$)	dM1
	$t = 0.306338477...$		
	$t = 0.306338477... \times 24 \times 60 = 441.1274082...$ minutes		
	$t = \underline{441\text{min}}$ or $t = \underline{7 \text{ hr } 21 \text{ mins}}$ (to nearest minute)	awrt $t = \underline{441}$ or $\underline{7 \text{ hr } 21 \text{ mins}}$	A1
			[3]
			14 marks

$\underline{P = P_0 e^{\sin \lambda t}}$ written down without the first M1 mark given scores all four marks in part (c).

Question Number	Scheme		Marks
	$\frac{dP}{dt} = kP$ and $t = 0,\ P = P_0$ (1)		
Aliter 8. (a) Way 2	$\int \frac{dP}{kP} = \int 1\, dt$	Separates the variables with $\int \frac{dP}{kP}$ and $\int dt$ on either side with integral signs not necessary.	M1
	$\frac{1}{k}\ln P = t;(+c)$	Must see $\frac{1}{k}\ln P$ and t; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \frac{1}{k}\ln P_0 = c$ $\left(\text{or } P = Ae^{kt} \Rightarrow P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\frac{1}{k}\ln P = t + \frac{1}{k}\ln P_0 \Rightarrow \ln P = kt + \ln P_0$ $\Rightarrow e^{\ln P} = e^{kt + \ln P_0} = e^{kt}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{kt}}$	$\underline{P = P_0 e^{kt}}$	A1 [4]
Aliter 8. (a) Way 3	$\int \frac{dP}{kP} = \int 1\, dt$	Separates the variables with $\int \frac{dP}{kP}$ and $\int dt$ on either side with integral signs not necessary.	M1
	$\frac{1}{k}\ln(kP) = t;(+c)$	Must see $\frac{1}{k}\ln(kP)$ and t; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \frac{1}{k}\ln(kP_0) = c$ $\left(\text{or } kP = Ae^{kt} \Rightarrow kP_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\frac{1}{k}\ln(kP) = t + \frac{1}{k}\ln(kP_0) \Rightarrow \ln(kP) = kt + \ln(kP_0)$ $\Rightarrow e^{\ln(kP)} = e^{kt + \ln(kP_0)} = e^{kt}.e^{\ln(kP_0)}$ $\Rightarrow kP = e^{kt}.(kP_0) \Rightarrow kP = kP_0 e^{kt}$ $\left(\text{or } kP = kP_0 e^{kt}\right)$		
	Hence, $\underline{P = P_0 e^{kt}}$	$\underline{P = P_0 e^{kt}}$	A1 [4]

Question Number	Scheme	Marks

	$\frac{dP}{dt} = \lambda P \cos \lambda t$ and $t = 0,\ P = P_0$ (1)		
Aliter 8. (c) Way 2	$\int \frac{dP}{\lambda P} = \int \cos \lambda t \ dt$	Separates the variables with $\int \frac{dP}{\lambda P}$ and $\int \cos \lambda t \, dt$ on either side with integral signs not necessary.	M1
	$\frac{1}{\lambda} \ln P = \frac{1}{\lambda} \sin \lambda t; (+c)$	Must see $\frac{1}{\lambda} \ln P$ and $\frac{1}{\lambda} \sin \lambda t$; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \ \Rightarrow\ \frac{1}{\lambda} \ln P_0 = c$ $\left(\text{or } P = Ae^{\sin \lambda t} \ \Rightarrow\ P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\frac{1}{\lambda} \ln P = \frac{1}{\lambda} \sin \lambda t + \frac{1}{\lambda} \ln P_0 \ \Rightarrow\ \ln P = \sin \lambda t + \ln P_0$		
	$\Rightarrow e^{\ln P} = e^{\sin \lambda t + \ln P_0} = e^{\sin \lambda t} . e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{\sin \lambda t}}$	$\underline{P = P_0 e^{\sin \lambda t}}$	A1 [4]

$\underline{P = P_0 e^{kt}}$ written down without the first M1 mark given scores all four marks in part (a).

$\underline{P = P_0 e^{\sin \lambda t}}$ written down without the first M1 mark given scores all four marks in part (c).

Question Number	Scheme		Marks
	$\frac{dP}{dt} = \lambda P \cos \lambda t$ and $t = 0,\ P = P_0$ (1)		
Aliter 8. (c) Way 3	$\int \frac{dP}{\lambda P} = \int \cos \lambda t \ dt$	Separates the variables with $\int \frac{dP}{\lambda P}$ and $\int \cos \lambda t \, dt$ on either side with integral signs not necessary.	M1
	$\frac{1}{\lambda}\ln(\lambda P) = \frac{1}{\lambda}\sin \lambda t; (+c)$	Must see $\frac{1}{\lambda}\ln(\lambda P)$ and $\frac{1}{\lambda}\sin \lambda t$; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \frac{1}{\lambda}\ln(\lambda P_0) = c$ $\left(\text{or } \lambda P = Ae^{\sin \lambda t} \Rightarrow \lambda P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\frac{1}{\lambda}\ln(\lambda P) = \frac{1}{\lambda}\sin \lambda t + \frac{1}{\lambda}\ln(\lambda P_0)$		
	$\Rightarrow \ln(\lambda P) = \sin \lambda t + \ln(\lambda P_0)$		
	$\Rightarrow e^{\ln(\lambda P)} = e^{\sin \lambda t + \ln(\lambda P_0)} = e^{\sin \lambda t}.e^{\ln(\lambda P_0)}$		
	$\Rightarrow \lambda P = e^{\sin \lambda t}.(\lambda P_0)$ $\left(\text{or } \lambda P = \lambda P_0 e^{\sin \lambda t}\right)$		
	Hence, $\underline{P = P_0 e^{\sin \lambda t}}$	$\underline{P = P_0 e^{\sin \lambda t}}$	A1
			[4]

- Note: dM1 denotes a method mark which is dependent upon the award of the previous method mark.
  ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.
  depM1$*$ denotes a method mark which is dependent upon the award of M1$*$.
  ft denotes "follow through"
  cao denotes "correct answer only"
  aef denotes "any equivalent form"