# June 2006
# 6666 Core Mathematics C4
# Mark Scheme

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **1.** | $6x - 4y\frac{dy}{dx} + 2 - 3\frac{dy}{dx} = 0$ | Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $\pm 3\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$.) | M1 |
| | | Correct equation. | A1 |
| | $\left\{\frac{dy}{dx} = \frac{6x+2}{4y+3}\right\}$ | *not necessarily required.* | |
| | At (0, 1), $\frac{dy}{dx} = \frac{0+2}{4+3} = \frac{2}{7}$ | Substituting x = 0 & y = 1 into an *equation* involving $\frac{dy}{dx}$; to give $\frac{2}{7}$ or $\frac{-2}{-7}$ | dM1;<br>A1 **cso** |
| | Hence m(**N**) = $-\frac{7}{2}$ or $\frac{-1}{\frac{2}{7}}$ | Uses m(**T**) to 'correctly' find m(**N**). Can be ft from "their tangent gradient". | A1√ oe. |
| | Either **N**: $y - 1 = -\frac{7}{2}(x - 0)$<br>or **N**: $y = -\frac{7}{2}x + 1$ | $y - 1 = m(x - 0)$ with 'their tangent or normal gradient';or uses $y = mx + 1$ with 'their tangent or normal gradient' ; | M1; |
| | **N**: $7x + 2y - 2 = 0$ | Correct equation in the form '$ax + by + c = 0$', where a, b and c are integers. | A1 oe **cso**<br>**[7]** |
| | | | **7 marks** |

**Beware:** $\frac{dy}{dx} = \frac{2}{7}$ does not necessarily imply the award of all the first four marks in this question.
So please ensure that you check candidates' initial differentiation before awarding the first A1 mark.

**Beware:** The final accuracy mark is for completely correct solutions. If a candidate flukes the final line then they must be awarded A0.

**Beware:** A candidate finding an m(**T**) = 0 can obtain A1ft for m(**N**) = ∞, but obtains M0 if they write $y - 1 = \infty(x - 0)$. If they write, however, **N**: $x = 0$, then can score M1.

**Beware:** A candidate finding an m(**T**) = ∞ can obtain A1ft for m(**N**) = 0, and also obtains M1 if they write $y - 1 = 0(x - 0)$ or $y = 1$.

**Beware:** The final **cso** refers to the whole question.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** | | | |
| **1.** | $6x\frac{dx}{dy} - 4y + 2\frac{dx}{dy} - 3 = 0$ | Differentiates implicitly to include either $\pm kx\frac{dx}{dy}$ or $\pm 2\frac{dx}{dy}$. (Ignore $\left(\frac{dx}{dy} = \right)$.) Correct equation. | M1 A1 |
| **Way 2** | $\left\{\frac{dx}{dy} = \frac{4y+3}{6x+2}\right\}$ | *not necessarily required.* | |
| | At (0, 1), $\frac{dx}{dy} = \frac{4+3}{0+2} = \frac{6}{2}$ | Substituting $x = 0$ & $y = 1$ into an *equation* involving $\frac{dx}{dy}$; to give $\frac{7}{2}$ | dM1; A1 **cso** |
| | Hence m(**N**) = $-\frac{7}{2}$ or $\frac{-1}{\frac{2}{7}}$ | Uses m(**T**) or $\frac{dx}{dy}$ to 'correctly' find m(**N**). Can be ft using "$-1.\frac{dx}{dy}$". | A1 √ oe. |
| | Either **N**: $y - 1 = -\frac{7}{2}(x - 0)$ or **N**: $y = -\frac{7}{2}x + 1$ | $y - 1 = m(x - 0)$ with 'their tangent, $\frac{dx}{dy}$ or normal gradient'; or uses $y = mx + 1$ with 'their tangent, $\frac{dx}{dy}$ or normal gradient'; | M1; |
| | **N**: $7x + 2y - 2 = 0$ | Correct equation in the form '$ax + by + c = 0$', where a, b and c are integers. | A1 oe **cso** |
| | | | **7 marks** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** **1. Way 3** | $2y^2 + 3y - 3x^2 - 2x - 5 = 0$ | | |
| | $\left(y+\frac{3}{4}\right)^2 - \frac{9}{16} = \frac{3x^2}{2} + x + \frac{5}{2}$ | | |
| | $y = \sqrt{\left(\frac{3x^2}{2}+x+\frac{49}{16}\right)} - \frac{3}{4}$ | | |
| | $\frac{dy}{dx} = \frac{1}{2}\left(\frac{3x^2}{2} + x + \frac{49}{16}\right)^{-\frac{1}{2}}(3x+1)$ | Differentiates using the chain rule;<br>Correct expression for $\frac{dy}{dx}$. | M1;<br>A1 oe |
| | At (0, 1), $\frac{dy}{dx} = \frac{1}{2}\left(\frac{49}{16}\right)^{-\frac{1}{2}} = \frac{1}{2}\left(\frac{4}{7}\right) = \frac{2}{7}$ | Substituting x = 0 into an *equation* involving $\frac{dy}{dx}$; to give $\frac{2}{7}$ or $\frac{-2}{-7}$ | dM1<br>A1 **cso** |
| | Hence m(**N**) = $-\frac{7}{2}$ | Uses m(**T**) to 'correctly' find m(**N**). Can be ft from "their tangent gradient". | A1$\sqrt{}$ |
| | Either **N**: $y - 1 = -\frac{7}{2}(x - 0)$<br>or **N**: y = $y = -\frac{2}{7}x + 1$ | $y - 1 = m(x - 0)$ with 'their tangent or normal gradient'; or uses $y = mx + 1$ with 'their tangent or normal gradient' | M1 |
| | **N**: $7x + 2y - 2 = 0$ | Correct equation in the form '$ax + by + c = 0$', where a, b and c are integers. | A1 oe |
| | | | **[7]** |
| | | | **7 marks** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **2.** (a) | $3x - 1 \equiv A(1-2x) + B$ | Considers this identity and either substitutes $x = \frac{1}{2}$, equates coefficients or solves simultaneous equations | *complete* M1 |
| | Let $x = \frac{1}{2}$; $\frac{3}{2} - 1 = B$ $\Rightarrow B = \frac{1}{2}$ | | |
| | Equate $x$ terms; $3 = -2A$ $\Rightarrow A = -\frac{3}{2}$ | $A = -\frac{3}{2}$; $B = \frac{1}{2}$ | A1;A1 |
| | (**No working seen**, but A and B correctly stated $\Rightarrow$ award all three marks. If one of A or B correctly stated give two out of the three marks available for this part.) | | **[3]** |
| (b) | $f(x) = -\frac{3}{2}(1-2x)^{-1} + \frac{1}{2}(1-2x)^{-2}$ | Moving powers to top on any one of the two expressions | M1 |
| | $= -\frac{3}{2}\left\{1 + (-1)(-2x); + \frac{(-1\right.$ | Either $1 \pm 2x$ or $1 \pm 4x$ or from either first or second expansions respectively | dM1; |

| | | |
|---|---|---|
| $+\frac{1}{2}\left\{1+(-2)(-2x);+\frac{(-2)}{2}\right.$ | Ignoring $-\frac{3}{2}$ and $\frac{1}{2}$, any one correct {.........}expansion. | A1 |
| | Both {.........} correct. | A1 |
| $=-\frac{3}{2}\left\{1+2x+4x^2+8x^3+\right.$ | | |
| $= -1 - x; + 0x^2 + 4x^3$ | $-1 - x; (0x^2) + 4x^3$ | A1; A1 |
| | | **[6]** |

**9 marks**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** **2.** (b) **Way 2** | $f(x) = (3x-1)(1-2x)^{-2}$ | Moving power to top | M1 |
| | $= (3x-1)\times\left(1+(-2)(-2x)\ ;+\frac{(-2)(-3)}{2!}(-2x)^2+\frac{(-2)(-3)(-4)}{3!}(-2x)^3+\ldots\right)$ | $1 \pm 4x$; Ignoring $(3x-1)$, correct (…….)expansion | dM1; A1 |
| | $= (3x-1)(1+4x+12x^2+32x^3+\ldots)$ | | |
| | $= 3x+12x^2+36x^3-1-4x-12x^2-32x^3+\ldots$ | Correct expansion | A1 |
| | $= -1-x;\ +0x^2+4x^3$ | $-1-x;\ (0x^2)+4x^3$ | A1; A1 |
| | | | **[6]** |
| ***Aliter*** **2.** (b) **Way 3** | Maclaurin expansion | | |
| | $f(x) = -\frac{3}{2}(1-2x)^{-1}+\frac{1}{2}(1-2x)^{-2}$ | Bringing bothpowers to top | M1 |
| | $f'(x) = -3(1-2x)^{-2}+2(1-2x)^{-3}$ | Differentiates to give $a(1-2x)^{-2} \pm b(1-2x)^{-3}$; $-3(1-2x)^{-2}+2(1-2x)^{-3}$ | M1; A1 oe |
| | $f''(x) = -12(1-2x)^{-3}+12(1-2x)^{-4}$ | | |
| | $f'''(x) = -72(1-2x)^{-4}+96(1-2x)^{-5}$ | Correct f”(x) and f```(x) | A1 |

$\therefore f(0) = -1$ , $f'(0) = -1$, $f''(0) = 0$ and $f'''(0) = 24$

gives $f(x) = -1 - x; + 0x^2 + 4x^3 + \ldots$ $-1 - x$ ; $(0x^2) + 4x^3$ A1; A1

**[6]**

| Question Number | Scheme | Marks |
| --- | --- | --- |
| ***Aliter*** **2.** (b) **Way 4** | $\mathrm{f}(x) = -3(2-4x)^{-1} + \frac{1}{2}(1-2x)^{-2}$ | Moving powers to top on any one of the two expressions M1 |
| | $= -3\left\{(2)^{-1} + (-1)(2)^{-2}(-4x); + \frac{(-1)(-2)}{2!}(2)^{-3}(-4x)^2 + \frac{(-1)(-2)(-3)}{3!}(2)^{-4}(-4x)^3 + \ldots\right\}$ | Either $\frac{1}{2} \pm x$ or $1 \pm 4x$ from either first or second expansions respectively dM1; |
| | $+\frac{1}{2}\left\{1 + (-2)(-2x); + \frac{(-2)(-3)}{2!}(-2x)^2 + \frac{(-2)(-3)(-4)}{3!}(-2x)^3 + \ldots\right\}$ | Ignoring $-3$ and $\frac{1}{2}$, any one correct{……..}expansion. A1<br>Both {……..} correct. A1 |
| | $= -3\left\{\frac{1}{2} + x + 2x^2 + 4x^3 + \ldots\right\} + \frac{1}{2}\left\{1 + 4x + 12x^2 + 32x^3 + \ldots\right\}$ | |
| | $= -1 - x\ ; + 0x^2 + 4x^3$ | $-1-x$ ; $(0x^2) + 4x^3$ A1; A1 |
| | | **[6]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **3.** (a) | Area Shaded = $\int_0^{2\pi} 3\sin\left(\frac{x}{2}\right) dx$ | | |
| | $= \left[\frac{-3\cos\left(\frac{x}{2}\right)}{\frac{1}{2}}\right]_0^{2\pi}$ | Integrating $3\sin\left(\frac{x}{2}\right)$ to give $k\cos\left(\frac{x}{2}\right)$ with $k \neq 1$. Ignore limits. | M1 |
| | $= \left[-6\cos\left(\frac{x}{2}\right)\right]_0^{2\pi}$ | $\mathbf{-6\cos}\left(\frac{x}{2}\right)$ or $\frac{-3}{\frac{1}{2}}\cos\left(\frac{x}{2}\right)$ | A1 oe. |
| | $= [-6(-1)] - [-6(1)] = 6 + 6 = \underline{12}$ | $\underline{12}$ | A1 cao |
| | | | **[3]** |
| | (Answer of 12 with no working scores M0A0A0.) | | |
| (b) | Volume $= \underline{\pi \int_0^{2\pi} \left(3\sin\left(\frac{x}{2}\right)\right)^2} \ dx = 9\pi \int_0^{2\pi} \sin^2\left(\frac{x}{2}\right) dx$ | **Use** of $\underline{V = \pi\int y^2 \ dx}$. Can be implied. Ignore limits. | M1 |
| | $\left[\text{NB}: \underline{\cos 2x = \pm 1 \pm 2\sin^2 x} \ \text{ gives } \sin^2 x = \frac{1-\cos 2x}{2}\right]$ <br> $\left[\text{NB}: \underline{\cos x = \pm 1 \pm 2\sin^2\left(\frac{x}{2}\right)} \ \text{ gives } \sin^2\left(\frac{x}{2}\right) = \frac{1-\cos x}{2}\right]$ | Consideration of the Half Angle Formula for $\sin^2\left(\frac{x}{2}\right)$ or the Double Angle Formula for $\sin^2 x$ | M1 * |
| | $\therefore \text{Volume} = 9(\pi)\int_0^{2\pi}\left(\frac{1-\cos x}{2}\right) dx$ | Correct expression for Volume <br> Ignore limits and $\pi$. | A1 |

$= \frac{9(\pi)}{2} \int_0^{2\pi} \underline{(1 - \cos x)}\, dx$

$= \frac{9(\pi)}{2} \left[\underline{x - \sin x}\right]_0^{2\pi}$ — Integrating to give $\underline{\pm ax \pm b\sin x}$; Correct integration $\underline{k - k\cos x \rightarrow kx - k\sin x}$ — depM1∗; A1

$= \frac{9\pi}{2}[(2\pi - 0) - (0 - 0)]$

$= \frac{9\pi}{2}(2\pi) = \underline{9\pi^2}$ or $\underline{88.8264\ldots}$ — Use of limits to give either $9\pi^2$ or awrt 88.8. Solution must be completely correct. No flukes allowed. — A1 **cso** **[6]**

**9 marks**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **4.** (a) | $x = \sin t, \quad y = \sin\left(t + \frac{\pi}{6}\right)$ | | |
| | $\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = \cos\left(t + \frac{\pi}{6}\right)$ | Attempt to differentiate both x and y wrt t to give two terms in cos<br>Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$ | M1 |

A1

When $t = \frac{\pi}{6}$,

$$\frac{dy}{dx} = \underline{\frac{\cos\left(\frac{\pi}{6}+\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58}$$

Divides in correct way and substitutes for t to give any of the four underlined oe: Ignore the double negative if candidate has differentiated $\sin \to -\cos$ — A1

When $t = \frac{\pi}{6}$, $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$

The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ or $\underline{\left(\frac{1}{2}, \text{awrt } 0.87\right)}$ — B1

**T**: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$

Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + "c"$. — dM1

Correct <u>EXACT</u> equation of <u>tangent</u> oe. — <u>A1</u> oe

or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$

or **T**: $\left[\underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}}\right]$

**[6]**

(b) $y = \sin\left(t + \frac{\pi}{6}\right) = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6}$

Use of compound angle formula for sine. — M1

$\text{Nb}: \sin^2 t + \cos^2 t \equiv 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t$

| | | |
|---|---|---|
| $\therefore x = \sin t$ gives $\cos t = \sqrt{(1 - x^2)}$ | Use of trig identity to find $\cos t$ in terms of x or $\cos^2 t$ in terms of x. | M1 |
| $\therefore y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t$ | | |
| gives $y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1 - x^2)}$ **AG** | Substitutes for $\sin t$, $\cos\frac{\pi}{6}$, $\cos t$ and $\sin\frac{\pi}{6}$ to give y in terms of x. | A1 cso |
| | | **[3]** |
| | | **9 marks** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** **4.** (a) **Way 2** | $x = \sin t,$ $y = \sin\left(t + \frac{\pi}{6}\right) = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6}$ | (Do not give this for part (b)) | |
| | | Attempt to differentiate x and y wrt t to give $\frac{dx}{dt}$ in terms of cos and $\frac{dy}{dt}$ in the form $\pm a\cos t \pm b\sin t$ | M1 |
| | $\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = \cos t \cos\frac{\pi}{6} - \sin t \sin\frac{\pi}{6}$ | Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$ | A1 |
| | When $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \underline{\frac{\cos\frac{\pi}{6}\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\sin\frac{\pi}{6}}{\cos\left(\frac{\pi}{6}\right)}}$ $= \underline{\frac{\frac{3}{4} - \frac{1}{4}}{\frac{\sqrt{3}}{2}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58}$ | Divides in correct way and substitutes for t to give any of the four underlined oe: | A1 |
| | When $t = \frac{\pi}{6}$, $x = \frac{1}{2}, \ y = \frac{\sqrt{3}}{2}$ | The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ or $\underline{\left(\frac{1}{2}, \text{awrt } 0.87\right)}$ | B1 |
| | **T**: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$ | Finding an equation of a tangent with their point and their tangent gradient or finds c and uses | dM1 |

$y = (\text{their gradient})x + "c"$. <u>A1</u> oe
Correct EXACT equation of <u>tangent</u> oe.

or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$

or **T**: $\left[\underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}}\right]$

**[6]**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** | | | |
| **4.** (a) | $y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1-x^2)}$ | | |
| **Way 3** | $\frac{dy}{dx} = \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1-x^2)^{-\frac{1}{2}}(-2x)$ | Attempt to differentiate two terms using the chain rule for the second term. | M1 |
| | | Correct $\frac{dy}{dx}$ | A1 |
| | $\underline{\frac{dy}{dx} = \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1-(0.5)^2)^{-\frac{1}{2}}(-2(0.5)) = \frac{1}{\sqrt{3}}}$ | Correct substitution of $x = \frac{1}{2}$ into a correct $\frac{dy}{dx}$ | A1 |
| | When $t = \frac{\pi}{6}$, $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$ | The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ or $\underline{\left(\frac{1}{2}, \text{awrt } 0.87\right)}$ | B1 |
| | **T**: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$ | Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + \text{"c"}$. | dM1 |
| | | Correct <u>EXACT</u> equation of <u>tangent</u> oe. | <u>A1</u> oe |
| | or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$ | | |
| | or **T**: $\left[\underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}}\right]$ | | |
| | | | **[6]** |
| ***Aliter*** | | | |

| | | | |
|---|---|---|---|
| **4.** (b) **Way 2** | $x = \sin t$ gives $y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\sqrt{(1 - \sin^2 t)}$ | Substitutes $x = \sin t$ into the equation give in y. | M1 |
| | $\text{Nb: } \sin^2 t + \cos^2 t \equiv 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t$ | | |
| | $\cos t = \sqrt{(1 - \sin^2 t)}$ | Use of trig identity to deduce that $\cos t = \sqrt{(1 - \sin^2 t)}$. | M1 |
| | gives $y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t$ | | |
| | Hence $y = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6} = \sin\left(t + \frac{\pi}{6}\right)$ | Using the compound angle formula to prove $y = \sin\left(t + \frac{\pi}{6}\right)$ | A1 cso |
| | | | **[3]** |
| | | | **9 marks** |

| Question Number | Scheme | | Marks |
| --- | --- | --- | --- |
| **5.** (a) | Equating **i** ; $0 = 6 + \lambda \Rightarrow \lambda = -6$ | $\underline{\lambda = -6}$ <br> Can be implied | B1 <br> $\Rightarrow$ d |
| | Using $\lambda = -6$ and | | |
| | equating **j** ; $a = 19 + 4(-6) = -5$ | For inserting **their stated** $\lambda$ into either a correct **j** or **k** component <br> Can be implied. | M1 $\Rightarrow$ d |
| | equating **k** ; $b = -1 - 2(-6) = 11$ | $a = -5$ and $b = 11$ | A1 <br> **[3]** |
| | With no working… <br> … only one of a or b stated correctly gains the first 2 marks. <br> … both a and b stated correctly gains 3 marks. | | |
| (b) | $\overrightarrow{OP} = (6+\lambda)\mathbf{i} + (19+4\lambda)\mathbf{j} + (-1-2\lambda)\mathbf{k}$ | | |
| | direction vector or $l_1 = \mathbf{d} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ | | |
| | $\overrightarrow{OP} \perp l_1 \Rightarrow \underline{\overrightarrow{OP} \bullet \mathbf{d} = 0}$ | Allow <u>this statement</u> for M1 if $\overrightarrow{OP}$ and $\mathbf{d}$ are defined as above. | |

| | | |
|---|---|---|
| ie. $\underline{\begin{pmatrix} 6+\lambda \\ 19+4\lambda \\ -1-2\lambda \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = 0}$ <br> $\left(\text{or } \underline{x+4y-2z=0}\right)$ | Allow either of these two <u>underlined statements</u> | M1 |
| $\therefore\ 6+\lambda+4(19+4\lambda)-2(-1-2\lambda) = 0$ | Correct equation | A1 oe |
| $6+\lambda+76+16\lambda+2+4\lambda = 0$ | Attempt to solve the equation in $\lambda$ | dM1 |
| $21\lambda+84=0 \Rightarrow \lambda=-4$ | $\lambda=-4$ | A1 |
| $\overrightarrow{OP} = (6-4)\mathbf{i} + (19+4(-4))\mathbf{j} + (-1-2(-4))\mathbf{k}$ | Substitutes their $\lambda$ into an expression for $\overrightarrow{OP}$ | M1 |
| $\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ | $2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ or $P(2, 3, 7)$ | A1 |
| | | **[6]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** <br> (b) <br> **Way 2** | $\overrightarrow{OP} = (6+\lambda)\mathbf{i} + (19+4\lambda)\mathbf{j} + (-1-2\lambda)\mathbf{k}$ | | |
| | $\overrightarrow{AP} = (6+\lambda-0)\mathbf{i} + (19+4\lambda+5)\mathbf{j} + (-1-2\lambda-11)\mathbf{k}$ | | |
| | direction vector or $l_1 = \mathbf{d} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ | | |
| | $\overrightarrow{AP} \perp \overrightarrow{OP} \Rightarrow \underline{\overrightarrow{AP} \bullet \overrightarrow{OP} = 0}$ | Allow <u>this statement</u> for M1 if $\overrightarrow{AP}$ and $\overrightarrow{OP}$ are defined as above. | |
| | ie. $\underline{\begin{pmatrix} 6+\lambda \\ 24+4\lambda \\ -12-2\lambda \end{pmatrix} \bullet \begin{pmatrix} 6+\lambda \\ 19+4\lambda \\ -1-2\lambda \end{pmatrix} = 0}$ | <u>underlined statement</u> | M1 |
| | $\therefore (6+\lambda)(6+\lambda) + (24+4\lambda)(19+4\lambda) + (-12-2\lambda)(-1-2\lambda) = 0$ | Correct equation | A1 oe |
| | $36+12\lambda+\lambda^2+456+96\lambda+76\lambda+16\lambda^2+12+24\lambda+2\lambda+4\lambda^2 = 0$ | Attempt to solve the equation in $\lambda$ | dM1 |
| | $21\lambda^2+210\lambda+504=0$ | | |
| | $\lambda^2+10\lambda+24=0 \Rightarrow (\lambda=-6)\ \underline{\lambda=-4}$ | $\lambda=-4$ | A1 |

| | | |
|---|---|---|
| $\overrightarrow{OP} = (6-4)\mathbf{i} + (19+4(-4))\mathbf{j} + (-1-2(-4))\mathbf{k}$ | Substitutes their $\lambda$ into an expression for $\overrightarrow{OP}$ | M1 |
| $\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ | $2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ or P(2, 3, 7) | A1 |
| | | **[6]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **5.** (c) | $\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ | | |
| | $\overrightarrow{OA} = 0\mathbf{i} - 5\mathbf{j} + 11\mathbf{k}$ and $\overrightarrow{OB} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}$ | | |
| | $\overrightarrow{AP} = \pm(2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k})$,<br>$\overrightarrow{PB} = \pm(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k})$<br>$\overrightarrow{AB} = \pm(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k})$ | Subtracting vectors to find any two of $\overrightarrow{AP}$, $\overrightarrow{PB}$ or $\overrightarrow{AB}$; and both are correctly ft using candidate's $\overrightarrow{OA}$ and $\overrightarrow{OP}$ found in parts (a) and (b) respectively. | M1;<br>A1$\sqrt{}$ ± |
| | As $\overrightarrow{AP} = \frac{2}{3}(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}) = \frac{2}{3}\overrightarrow{PB}$<br>or $\overrightarrow{AB} = \frac{5}{2}(2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}) = \frac{5}{2}\overrightarrow{AP}$<br>or $\overrightarrow{AB} = \frac{5}{3}(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}) = \frac{5}{3}\overrightarrow{PB}$<br>or $\overrightarrow{PB} = \frac{3}{2}(2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}) = \frac{3}{2}\overrightarrow{AP}$<br>or $\overrightarrow{AP} = \frac{2}{5}(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}) = \frac{2}{5}\overrightarrow{AB}$<br>or $\overrightarrow{PB} = \frac{3}{5}(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}) = \frac{3}{5}\overrightarrow{AB}$ etc… | $\overrightarrow{AP} = \frac{2}{3}\overrightarrow{PB}$<br>or $\overrightarrow{AB} = \frac{5}{2}\overrightarrow{AP}$<br>or $\overrightarrow{AB} = \frac{5}{3}\overrightarrow{PB}$<br>or $\overrightarrow{PB} = \frac{3}{2}\overrightarrow{AP}$<br>or $\overrightarrow{AP} = \frac{2}{5}\overrightarrow{AB}$<br>or $\overrightarrow{PB} = \frac{3}{5}\overrightarrow{AB}$ | |
| | alternatively candidates could say for example that<br>$\overrightarrow{AP} = 2(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$ $\overrightarrow{PB} = 3(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$ | | |
| | then <u>the points A, P and B are collinear</u>. | <u>A, P and B are collinear</u><br>Completely correct proof. | A1 |
| | $\therefore \overrightarrow{AP} : \overrightarrow{PB} = 2:3$ | 2:3 or $1:\frac{3}{2}$ or $\sqrt{84}:\sqrt{189}$ aef | B1 oe |

| | | | |
|---|---|---|---|
| | | allow SC $\frac{2}{3}$ | **[4]** |
| ***Aliter*** **5.** (c) Way 2 | At B; $\underline{5 = 6 + \lambda}$ , $\underline{15 = 19 + 4\lambda}$ or $\underline{1 = -1 - 2\lambda}$<br>or at B; $\lambda = -1$ | Writing down any of the three underlined equations. | M1 |
| | gives $\lambda = -1$ for all three equations.<br>or when $\lambda = -1$, this gives $\mathbf{r} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}$ | $\lambda = -1$ for all three equations or $\lambda = -1$ gives $\mathbf{r} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}$ | A1 |
| | Hence B lies on $l_1$. As stated in the question both A and P lie on $l_1$. $\therefore$ A, P and B are collinear. | Must state B lies on $l_1$ $\Rightarrow$ A, P and B are collinear | A1 |
| | $\therefore \overrightarrow{AP} : \overrightarrow{PB} = 2:3$ | 2:3 or aef | B1 oe |
| | | | **[4]** |
| | | | **13 marks** |

| Question Number | Scheme | | Marks | |
|---|---|---|---|---|
| **6.** (a) | | | | |

| x | 1 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|
| y | 0 | 0.5 ln 1.5 | ln 2 | 1.5 ln 2.5 | 2 ln 3 |
| or y | 0 | 0.2027325541 … | ln2 | 1.374436098 … | 2 ln 3 |

| | | | | |
|---|---|---|---|---|
| | | Either 0.5 ln 1.5 and 1.5 ln 2.5 or awrt 0.20 and 1.37 (or mixture of decimals and ln's) | B1 | **[1]** |
| (b)(i) | $I_1 \approx \frac{1}{2} \times 1 \times \underline{\{0 + 2(\ln 2) + 2\ln 3\}}$ | <u>For structure of trapezium rule</u> $\underline{\{............\}}$ ; | M1; | |
| | $= \frac{1}{2} \times 3.583518938... = 1.791759... = 1.792$ (4sf) | 1.792 | A1 cao | |
| (ii) | $I_2 \approx \frac{1}{2} \times 0.5\ ; \times \underline{\{0 + 2(0.5\ln 1.5 + \ln 2 + 1.5\ln 2.5) + 2\ln 3\}}$ | Outside brackets $\frac{1}{2} \times 0.5$ | B1; | |
| | | <u>For structure of trapezium rule</u> $\underline{\{............\}}$ ; | M1 $\sqrt{}$ | |
| | $= \frac{1}{4} \times 6.737856242... = 1.684464...$ | awrt 1.684 | A1 | **[5]** |
| (c) | With increasing ordinates, <u>the line segments at the top of the trapezia are closer to the curve.</u> | <u>Reason</u> or an appropriate diagram elaborating the correct | B1 | |

reason. **[1]**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **6.** (d) | $\begin{cases} u = \ln x & \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x - 1 & \Rightarrow v = \frac{x^2}{2} - x \end{cases}$ | Use of 'integration by parts' formula in the correct direction | |
| | $I = \left(\frac{x^2}{2} - x\right)\ln x - \int \frac{1}{x}\left(\frac{x^2}{2} - x\right) dx$ | Correct expression | |
| | $= \left(\frac{x^2}{2} - x\right)\ln x - \underline{\int\left(\frac{x}{2} - 1\right) dx}$ | An attempt to multiply at least one term through by $\frac{1}{x}$ and an attempt to ... | |
| | $= \left(\frac{x^2}{2} - x\right)\ln x - \underline{\left(\frac{x^2}{4} - x\right)} \quad (+c)$ | … integrate; <br> <u>correct integration</u> | |
| | $\therefore I = \left[\left(\frac{x^2}{2} - x\right)\ln x - \frac{x^2}{4} + x\right]_1^3$ | | |
| | $= \left(\frac{3}{2}\ln 3 - \frac{9}{4} + 3\right) - \left(-\frac{1}{2}\ln 1 - \frac{1}{4} + 1\right)$ | Substitutes limits of 3 and 1 and subtracts. | |
| | $= \frac{3}{2}\ln 3 + \frac{3}{4} + 0 - \frac{3}{4} \quad = \underline{\frac{3}{2}\ln 3}$ **AG** | $\frac{3}{2}\ln 3$ ↄ | |
| | | | **[6]** |

***Aliter***

**6.** (d)

**Way 2**

| | |
|---|---|
| $\int (x-1)\ln x\,dx = \int x\ln x\,dx - \int \ln x\,dx$ | |
| $\int x\ln x\,dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}.\left(\frac{1}{x}\right)dx$ | Correct application of 'by parts' |
| $= \frac{x^2}{2}\ln x - \frac{x^2}{4}$ (+ c) | Correct integration |
| $\int \ln x\,dx = x\ln x - \int x.\left(\frac{1}{x}\right)dx$ | Correct application of 'by parts' |
| $= x\ln x - x$ (+ c) | Correct integration |
| $\therefore \int_1^3 (x-1)\ln x\,dx = \left(\frac{9}{2}\ln 3 - 2\right) - \left(3\ln 3 - 2\right) = \frac{3}{2}\ln 3$ | Substitutes limits of 3 and 1 into both integrands and subtracts. |
| **AG** | $\frac{3}{2}\ln 3$ |

**[6]**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** | | | |
| **6.** (d) | $\left\{\begin{array}{ll} u = \ln x & \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = (x-1) & \Rightarrow v = \frac{(x-1)^2}{2} \end{array}\right\}$ | Use of 'integration by parts' formula in the correct direction | |
| **Way 3** | | | |
| | $I = \frac{(x-1)^2}{2}\ln x - \int \frac{(x-1)^2}{2x}\,dx$ | Correct expression | |
| | $= \frac{(x-1)^2}{2}\ln x - \int \frac{x^2 - 2x + 1}{2x}\,dx$ | Candidate multiplies out numerator to obtain three terms… | |
| | $= \frac{(x-1)^2}{2}\ln x - \int \underline{\left(\frac{1}{2}x - 1 + \frac{1}{2x}\right)}\,dx$ | … multiplies at least one term through by $\frac{1}{x}$ and then attempts to ... | |
| | $= \frac{(x-1)^2}{2}\ln x - \underline{\left(\frac{x^2}{4} - x + \frac{1}{2}\ln x\right)}$ (+c) | … integrate the result; <br> <u>correct integration</u> | |
| | $\therefore I = \left[\frac{(x-1)^2}{2}\ln x - \frac{x^2}{4} + x - \frac{1}{2}\ln x\right]_1^3$ | | |
| | $= \left(2\ln 3 - \frac{9}{4} + 3 - \frac{1}{2}\ln 3\right) - \left(0 - \frac{1}{4} + 1 - 0\right)$ | Substitutes limits of 3 and 1 and subtracts. | |
| | $= 2\ln 3 - \frac{1}{2}\ln 3 + \frac{3}{4} + \frac{1}{4} - 1 \quad = \underline{\frac{3}{2}\ln 3}$ **AG** | $\frac{3}{2}\ln 3$ | |

[6]

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** **6.** (d) **Way 4** | By substitution $u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x}$ | | |
| | $I = \int (e^u - 1).ue^u \; du$ | Correct expression | |
| | $= \int u(e^{2u} - e^u)du$ | Use of 'integration by parts' formula in the correct direction | M1 |
| | $= u\left(\frac{1}{2}e^{2u} - e^u\right) - \int \underline{\left(\frac{1}{2}e^{2u} - e^u\right)} dx$ | Correct expression | A1 |
| | $= u\left(\frac{1}{2}e^{2u} - e^u\right) - \underline{\left(\frac{1}{4}e^{2u} - e^u\right)}$ (+c) | Attempt <u>to integrate</u>; <u>correct integration</u> | M1; A1 |
| | $\therefore I = \left[\frac{1}{2}ue^{2u} - ue^u - \frac{1}{4}e^{2u} + e^u\right]_{\ln 1}^{\ln 3}$ | | |
| | $= \left(\frac{9}{2}\ln 3 - 3\ln 3 - \frac{9}{4} + 3\right) - \left(0 - 0 - \frac{1}{4} + 1\right)$ | Substitutes limits of ln3 and ln1 and subtracts. | ddM1 |
| | $= \frac{3}{2}\ln 3 + \frac{3}{4} + \frac{1}{4} - 1 \quad = \underline{\frac{3}{2}\ln 3}$ **AG** | $\frac{3}{2}\ln 3$ | A1 cso |
| | | | **[6]** |
| | | | **13** |

**marks**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **7.** (a) | From question, $\frac{dS}{dt} = 8$ | $\frac{dS}{dt} = 8$ | B1 |
| | $S = 6x^2 \Rightarrow \frac{dS}{dx} = 12x$ | $\frac{dS}{dx} = 12x$ | B1 |
| | $\frac{dx}{dt} = \frac{dS}{dt} \div \frac{dS}{dx} = \underline{\frac{8}{12x}}; = \frac{\frac{2}{3}}{x} \Rightarrow \left(k = \tfrac{2}{3}\right)$ | Candidate's $\frac{dS}{dt} \div \frac{dS}{dx}$; $\underline{\frac{8}{12x}}$ | M1; <u>A1</u>oe |
| | | | **[4]** |
| (b) | $V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$ | $\frac{dV}{dx} = 3x^2$ | B1 |
| | $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 3x^2.\left(\frac{2}{3x}\right); = 2x$ | Candidate's $\frac{dV}{dx} \times \frac{dx}{dt}$; $\lambda x$ | M1; A1$\sqrt{}$ |
| | As $x = V^{\frac{1}{3}}$, then $\frac{dV}{dt} = 2V^{\frac{1}{3}}$ **AG** | Use of $x = V^{\frac{1}{3}}$, to give $\frac{dV}{dt} = 2V^{\frac{1}{3}}$ | A1 |
| | | | **[4]** |
| (c) | $\int \frac{dV}{V^{\frac{1}{3}}} = \int 2\ dt$ | Separates the variables with $\int \frac{dV}{V^{\frac{1}{3}}}$ or $\int V^{-\frac{1}{3}} dV$ on one side and $\int 2\ dt$ on the other side. integral signs not necessary. | B1 |

| | | |
|---|---|---|
| $\int V^{-\frac{1}{3}}\, dV = \int 2\, dt$ | | |
| $\frac{3}{2}V^{\frac{2}{3}} = 2t \ \ (+c)$ | Attempts to integrate and … … must see $V^{\frac{2}{3}}$ and 2t; Correct equation with/without + c. | M1; A1 |
| $\frac{3}{2}(8)^{\frac{2}{3}} = 2(0) + c \ \Rightarrow\ c = 6$ | Use of V = 8 and t = 0 in a changed equation containing c ; $c = 6$ | M1∗; A1 |
| Hence: $\frac{3}{2}V^{\frac{2}{3}} = 2t + 6$ | | |
| $\frac{3}{2}\left(16\sqrt{2}\right)^{\frac{2}{3}} = 2t + 6 \quad \Rightarrow \quad 12 = 2t + 6$ | Having found their "c" candidate … … substitutes $V = 16\sqrt{2}$ into an equation involving V, t and "c". | depM1 ∗ |
| giving t = 3. | t = 3 | A1 cao **[7]** |
| | | **15 marks** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** | | | |
| **7.** (b) | $x = V^{\frac{1}{3}}$ & $S = 6x^2 \Rightarrow S = 6V^{\frac{2}{3}}$ | $S = 6V^{\frac{2}{3}}$ | B1 $\sqrt{}$ |
| **Way 2** | | | |
| | $\frac{dS}{dV} = 4V^{-\frac{1}{3}}$ or $\frac{dV}{dS} = \frac{1}{4}V^{\frac{1}{3}}$ | $\frac{dS}{dV} = 4V^{-\frac{1}{3}}$ or $\frac{dV}{dS} = \frac{1}{4}V^{\frac{1}{3}}$ | B1 |
| | $\frac{dV}{dt} = \frac{dS}{dt} \times \frac{dV}{dS} = 8.\left(\frac{1}{4V^{-\frac{1}{3}}}\right); = \frac{2}{V^{-\frac{1}{3}}} = 2V^{\frac{1}{3}}$ **AG** | Candidate's $\frac{dS}{dt} \times \frac{dV}{dS}$; $2V^{\frac{1}{3}}$ | M1; A1 |
| | | | **[4]** |
| ***Aliter*** | | | |
| **7.** (c) | $\int \frac{dV}{2V^{\frac{1}{3}}} = \int 1\, dt$ | Separates the variables with $\int \frac{dV}{2V^{\frac{1}{3}}}$ or $\int \frac{1}{2}V^{-\frac{1}{3}}dV$ oe on one side and $\int 1\, dt$ on the other side. integral signs not necessary. | B1 |
| **Way 2** | $\frac{1}{2}\int V^{-\frac{1}{3}}\, dV = \int 1\, dt$ | | |
| | $\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)V^{\frac{2}{3}} = t \quad (+c)$ | Attempts to integrate and … … must see $V^{\frac{2}{3}}$ and t; Correct equation with/without + c. | M1; A1 |
| | $\frac{3}{4}(8)^{\frac{2}{3}} = (0) + c \Rightarrow c = 3$ | Use of V = 8 and t = 0 in a changed equation containing c ; $c = 3$ | M1∗; A1 |
| | Hence: $\frac{3}{4}V^{\frac{2}{3}} = t + 3$ | Having found their "c" candidate … | |

| | | |
|---|---|---|
| $\frac{3}{4}\left(16\sqrt{2}\right)^{\frac{2}{3}} = t + 3 \quad \Rightarrow \quad 6 = t + 3$ | … substitutes $V = 16\sqrt{2}$ into an equation involving V, t and "c". | depM1∗ |
| giving t = 3. | t = 3 | A1 cao |
| | | **[7]** |

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| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** | *similar to way 1.* | | |
| (b) | $V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$ | $\frac{dV}{dx} = 3x^2$ | B1 |
| **Way 3** | | | |
| | $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dS}{dt} \times \frac{dx}{dS} = 3x^2.8.\left(\frac{1}{12x}\right); = 2x$ | Candidate's $\frac{dV}{dx} \times \frac{dS}{dt} \times \frac{dx}{dS}$; $\lambda x$ | M1; A1$\sqrt{}$ |
| | As $x = V^{\frac{1}{3}}$, then $\frac{dV}{dt} = 2V^{\frac{1}{3}}$ **AG** | Use of $x = V^{\frac{1}{3}}$, to give $\frac{dV}{dt} = 2V^{\frac{1}{3}}$ | A1 |
| | | | **[4]** |
| ***Aliter*** | | | |
| (c) | $\int \frac{dV}{V^{\frac{1}{3}}} = \int 2\ dt$ | Separates the variables with $\int \frac{dV}{V^{\frac{1}{3}}}$ or $\int V^{-\frac{1}{3}} dV$ on one side and $\int 2\ dt$ on the other side. integral signs not necessary. | B1 |
| **Way 3** | $\int V^{-\frac{1}{3}}\ dV = \int 2\ dt$ | | |
| | $V^{\frac{2}{3}} = \frac{4}{3}t$ (+c) | Attempts to integrate and … … must see $V^{\frac{2}{3}}$ and $\frac{4}{3}t$; Correct equation with/without + c. | M1; A1 |
| | $(8)^{\frac{2}{3}} = \frac{4}{3}(0) + c \Rightarrow c = 4$ | Use of V = 8 and t = 0 in a changed equation containing c ; $c = 4$ | M1*; A1 |
| | Hence: $V^{\frac{2}{3}} = \frac{4}{3}t + 4$ | | |

| | | |
|---|---|---|
| | Having found their "c" candidate … | |
| $\left(16\sqrt{2}\right)^{\frac{2}{3}} = \frac{4}{3}t + 6 \quad \Rightarrow \quad 8 = \frac{4}{3}t + 4$ | … substitutes $V = 16\sqrt{2}$ into an equation involving V, t and "c". | depM1 * |
| giving t = 3. | t = 3 | A1 cao **[7]** |