

GCE Examinations
Advanced Subsidiary

Core Mathematics C3

Paper L

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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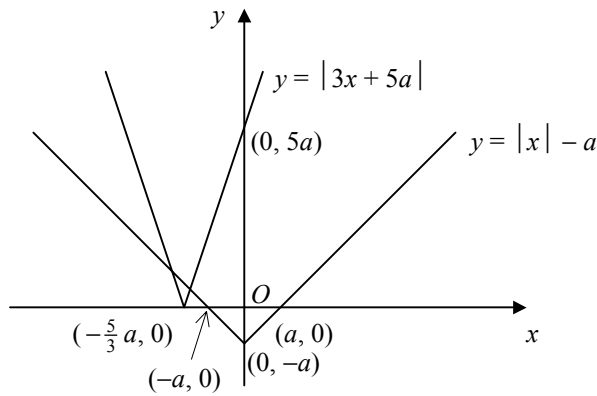
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C3 Paper L – Marking Guide

1.	<p>(a) $f(x) = \frac{2(x-2)+1}{x-2} = 2 + \frac{1}{x-2}$ $x > 2 \therefore f(x) > 2$</p> <p>(b) $ff(x) = f\left(\frac{2x-3}{x-2}\right) = \frac{2\left(\frac{2x-3}{x-2}\right)-3}{\frac{2x-3}{x-2}-2}$ $= \frac{2(2x-3)-3(x-2)}{(2x-3)-2(x-2)} = \frac{4x-6-3x+6}{2x-3-2x+4} = x$</p> <p>(c) $f^{-1}(x) = \frac{2x-3}{x-2}$</p>	M1 A1 M1 M1 A1 B1	(6)
<hr/>			
2.	<p>(a) $4x - 3 = \ln 2$ $x = \frac{1}{4}(3 + \ln 2)$</p> <p>(b) $\ln(2y - 1) - \ln(3 - y) = \ln \frac{2y-1}{3-y} = 1$ $\frac{2y-1}{3-y} = e$ $2y - 1 = e(3 - y), \quad y(e + 2) = 3e + 1$ $y = \frac{3e+1}{e+2}$</p>	M1 M1 A1 M1 A1 M1 A1	(7)
<hr/>			
3.	<p>(a) $\frac{dy}{dx} = 2e^x - \frac{6}{x}$ $x = 1, y = 2e, \text{ grad} = 2e - 6$ $\therefore y - 2e = (2e - 6)(x - 1) \quad [y = (2e - 6)x + 6]$</p> <p>(b) $x = 0 \Rightarrow y = 6$ $y = 0 \Rightarrow (2e - 6)x + 6 = 0$ $x = \frac{-6}{2e-6} = \frac{3}{3-e}$ $\text{area} = \frac{1}{2} \times 6 \times \frac{3}{3-e} = \frac{9}{3-e}$</p>	M1 A1 M1 A1 M1 A1 M1 A1	(8)
<hr/>			
4.	<p>(a) $= \frac{(x-10)(2x-1) - (x-8)(x+4)}{(x-3)(x+4)(2x-1)}$ $= \frac{x^2 - 17x + 42}{(x-3)(x+4)(2x-1)}$ $= \frac{(x-14)(x-3)}{(x-3)(x+4)(2x-1)} = \frac{x-14}{(x+4)(2x-1)}$</p> <p>(b) $\frac{x-14}{(x+4)(2x-1)} = 1, \quad x - 14 = 2x^2 + 7x - 4$ $x^2 + 3x + 5 = 0$ $b^2 - 4ac = 9 - 20 = -11$ $b^2 - 4ac < 0 \therefore \text{no real roots}$</p>	M1 A1 A1 M1 A1 M1 A1 M1 A1	(9)
<hr/>			
5.	<p>$\frac{\tan x + \tan 45}{1 - \tan x \tan 45} - \tan x = 4$ $\frac{\tan x + 1}{1 - \tan x} = 4 + \tan x$ $\tan x + 1 = (4 + \tan x)(1 - \tan x)$ $\tan^2 x + 4 \tan x - 3 = 0$ $\tan x = \frac{-4 \pm \sqrt{16+12}}{2} = -2 \pm \sqrt{7}$ $x = 180 - 77.9, -77.9 \text{ or } 32.9, -180 + 32.9$ $x = -147.1, -77.9, 32.9, 102.1 \text{ (1dp)}$</p>	M1 A1 M1 A1 M1 B1 M1 A2	(9)

6. (a)



B3

B3

(b) $-x - a = 3x + 5a \Rightarrow x = -\frac{3}{2}a$ M1 A1

$-x - a = -(3x + 5a) \Rightarrow x = -2a, \quad x = -2a, -\frac{3}{2}a$ M1 A1 (10)

7. (a) $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$

let $A = B = \frac{x}{2}$ $\cos x \equiv \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}$ M1

$\cos x \equiv (1 - \sin^2 \frac{x}{2}) - \sin^2 \frac{x}{2}$ M1

$\cos x \equiv 1 - 2 \sin^2 \frac{x}{2}$ A1

(b) $\text{LHS} \equiv \frac{1 - (1 - 2 \sin^2 \frac{x}{2})}{2 \sin \frac{x}{2} \cos \frac{x}{2}}$ M1

$\equiv \frac{2 \sin^2 \frac{x}{2}}{2 \sin \frac{x}{2} \cos \frac{x}{2}} \equiv \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} \equiv \tan \frac{x}{2} \equiv \text{RHS}$ M1 A1

(c) $\tan \frac{x}{2} = 2 \sec^2 \frac{x}{2} - 5, \quad \tan \frac{x}{2} = 2(1 + \tan^2 \frac{x}{2}) - 5$ M1

$2 \tan^2 \frac{x}{2} - \tan \frac{x}{2} - 3 = 0, \quad (2 \tan \frac{x}{2} - 3)(\tan \frac{x}{2} + 1) = 0$ M1

$\tan \frac{x}{2} = -1$ or $\frac{3}{2}$ A1

$\frac{x}{2} = 135$ or 56.310 B1

$x = 112.6^\circ$ (1dp), 270° A2 (12)

8. (a) $\frac{dy}{dx} = 2 \times e^{-x} + (2x + 3) \times (-e^{-x}) = -(2x + 1)e^{-x}$ M1 A1

SP: $-(2x + 1)e^{-x} = 0$
 $x = -\frac{1}{2} \therefore (-\frac{1}{2}, 2e^{\frac{1}{2}})$ M1 A1

(b) $x = 0, y = 3, \text{ grad} = -1, \text{ grad of normal} = 1$ M1
 $\therefore y = x + 3$ A1

(c) $x + 3 = (2x + 3)e^{-x}$
 $x + 3 - (2x + 3)e^{-x} = 0$ M1

let $f(x) = x + 3 - (2x + 3)e^{-x}$
 $f(-2) = 8.4, f(-1) = -0.72$ M1
 sign change, $f(x)$ continuous \therefore root A1

(d) $x_1 = -1.1619, x_2 = -1.2218, x_3 = -1.2408, x_4 = -1.2465 = -1.25$ (2dp) M1 A2

(e) $f(-1.255) = 0.026, f(-1.245) = -0.016$ M1
 sign change, $f(x)$ continuous \therefore root A1 (14)

Total (75)

Performance Record – C3 Paper L

Question no.	1	2	3	4	5	6	7	8	Total
Topic(s)	functions	exponentials and logarithms	differentiation	rational expressions	trigonometry	functions	trigonometry	differentiation, numerical methods	
Marks	6	7	8	9	9	10	12	14	75
Student									