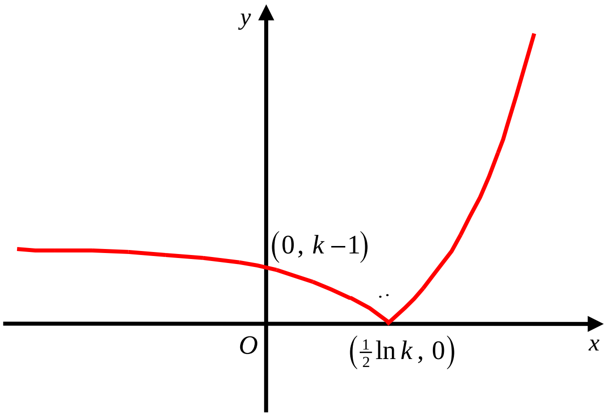
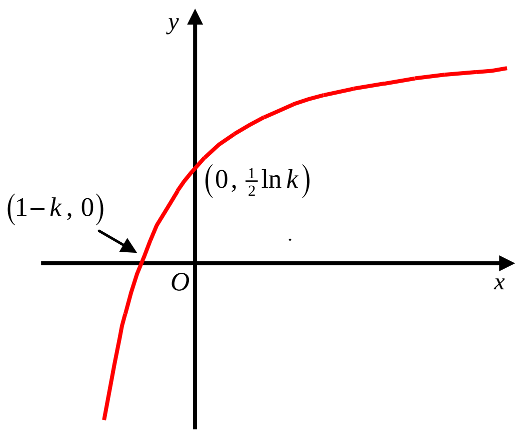
Question Number	Scheme	Marks
1. (a)	Iterative formula: $x_{n+1} = \frac{2}{(x_n)^2} + 2$, $x_0 = 2.5$	
	$x_1 = \frac{2}{(2.5)^2} + 2$	M1
	$x_1 = 2.32$, $x_2 = 2.371581451...$	A1
	$x_3 = 2.355593575...$, $x_4 = 2.360436923...$	A1 cso (3)
(b)	Let $f(x) = -x^3 + 2x^2 + 2 = 0$	
	$f(2.3585) = 0.00583577...$	M1
	$f(2.3595) = -0.00142286...$	M1
	Sign change (and $f(x)$ is continuous) therefore a root α is such that $\alpha \in (2.3585, 2.3595) \Rightarrow \alpha = 2.359$ (3 dp)	A1 (3)
		(6 marks)
2. (a)	$\cos^2\theta + \sin^2\theta = 1$ $\left(\div \cos^2\theta\right)$	
	$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$	M1
	$1 + \tan^2\theta = \sec^2\theta$	
	$\tan^2\theta = \sec^2\theta - 1$ (as required)	A1 cso (2)
(b)	$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2$, (eqn *) $0 \le \theta < 360^\circ$	
	$2(\sec^2\theta - 1) + 4\sec\theta + \sec^2\theta = 2$	M1
	$2\sec^2\theta - 2 + 4\sec\theta + \sec^2\theta = 2$	
	$3\sec^2\theta + 4\sec\theta - 4 = 0$	M1
	$(\sec\theta + 2)(3\sec\theta - 2) = 0$	M1
	$\sec\theta = -2$ or $\sec\theta = \frac{2}{3}$	
	$\frac{1}{\cos\theta} = -2$ or $\frac{1}{\cos\theta} = \frac{2}{3}$	
	$\cos\theta = -\frac{1}{2}$; or $\cos\theta = \frac{3}{2}$	A1;
	$\alpha = 120^\circ$ or $\alpha =$ no solutions	
	$\theta_1 = 120^\circ$	A1
	$\theta_2 = 240^\circ$	B1 ft (6)
		(8 marks)

Question Number	Scheme	Marks
3. (a)	$P = 80\mathrm{e}^{\frac{t}{5}}$	
	$t = 0 \Rightarrow P = 80\mathrm{e}^{\frac{0}{5}} = 80(1) = \underline{80}$	B1 (1)
(b)	$P = 1000 \Rightarrow 1000 = 80\mathrm{e}^{\frac{t}{5}} \Rightarrow \frac{1000}{80} = \mathrm{e}^{\frac{t}{5}}$	M1
	$\therefore\ t = 5\ln\left(\frac{1000}{80}\right)$	
	$t = 12.6286...$	A1 (2)
(c)	$\frac{\mathrm{d}P}{\mathrm{d}t} = 16\mathrm{e}^{\frac{t}{5}}$	M1 A1 (2)
(d)	$50 = 16\mathrm{e}^{\frac{t}{5}}$	
	$\therefore\ t = 5\ln\left(\frac{50}{16}\right)$ $\{= 5.69717...\}$	M1
	$P = 80\mathrm{e}^{\frac{1}{5}\left(5\ln\left(\frac{50}{16}\right)\right)}$ or $P = 80\mathrm{e}^{\frac{1}{5}(5.69717...)}$	M1
	$P = \frac{80(50)}{16} = \underline{250}$	A1 (3)
		(8 marks)

Question Number	Scheme	Marks
4. (i)(a)	$y = x^2 \cos 3x$	
	Apply product rule: $\left\{ \begin{matrix} u = x^2 & v = \cos 3x \\ \frac{du}{dx} = 2x & \frac{dv}{dx} = -3\sin 3x \end{matrix} \right\}$	M1
	$\frac{dy}{dx} = 2x\cos 3x - 3x^2 \sin 3x$	A1 A1 (3)
(i)(b)	$y = \frac{\ln(x^2+1)}{x^2+1}$	
	$u = \ln(x^2+1) \Rightarrow \frac{du}{dx} = \frac{2x}{x^2+1}$	M1 A1
	Apply quotient rule: $\left\{ \begin{matrix} u = \ln(x^2+1) & v = x^2+1 \\ \frac{du}{dx} = \frac{2x}{x^2+1} & \frac{dv}{dx} = 2x \end{matrix} \right\}$	M1
	$\frac{dy}{dx} = \frac{\left(\frac{2x}{x^2+1}\right)(x^2+1) - 2x\ln(x^2+1)}{(x^2+1)^2}$	A1
(ii)	$y = \sqrt{4x+1},\ x > -\frac{1}{4}$	
	At P, $y = \sqrt{4(2)+1} = \underline{\sqrt{9}} = \underline{3}$	B1
	$\frac{dy}{dx} = \underline{\frac{1}{2}(4x+1)^{-\frac{1}{2}}(4)}$	M1
	$\frac{dy}{dx} = \frac{2}{(4x+1)^{\frac{1}{2}}}$	A1
	At P, $\frac{dy}{dx} = \frac{2}{(4(2)+1)^{\frac{1}{2}}}$	M1
	Hence m(**T**) = $\frac{2}{3}$	
	Either **T**: $y - 3 = \frac{2}{3}(x-2)$;	M1
	T: $3y - 9 = 2(x-2)$;	
	T: $3y - 9 = 2x - 4$	
	T: $\underline{2x - 3y + 5 = 0}$	A1 (6)
		(13 marks)

Question Number	Scheme		Marks
5. (a)		Curve retains shape when $x > \tfrac{1}{2}\ln k$	B1
		Curve reflects through the x-axis when $x < \tfrac{1}{2}\ln k$	B1
		$(0,\ k-1)$ and $\left(\tfrac{1}{2}\ln k,\ 0\right)$ marked in the correct positions.	B1 (3)
(b)		Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote)	B1
		$(1-k,\ 0)$ and $\left(0,\ \tfrac{1}{2}\ln k\right)$	B1 (2)
(c)	Range of f: $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k,\ \infty)}$		B1 (1)
(d)	$y = e^{2x} - k \Rightarrow y + k = e^{2x}$		M1
	$\Rightarrow \ln(y+k) = 2x$		
	$\Rightarrow \tfrac{1}{2}\ln(y+k) = x$		M1
	Hence $f^{-1}(x) = \underline{\tfrac{1}{2}\ln(x+k)}$		A1 cao (3)
	$f^{-1}(x)$: Domain: $\underline{x > -k}$ or $\underline{(-k,\ \infty)}$		B1ft (1)
			(10 marks)

Question Number	Scheme	Marks
6. (a)	$A = B \Rightarrow \cos(A+A) = \cos 2A = \underline{\cos A\cos A - \sin A\sin A}$	M1
	$\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A + \sin^2 A = 1$ gives	
	$\underline{\cos 2A} = 1 - \sin^2 A - \sin^2 A = \underline{1 - 2\sin^2 A}$ (as required)	A1 (2)
(b)	$C_1 = C_2 \Rightarrow 3\sin 2x = 4\sin^2 x - 2\cos 2x$	M1
	$3\sin 2x = 4\left(\frac{1-\cos 2x}{2}\right) - 2\cos 2x$	M1
	$3\sin 2x = 2(1-\cos 2x) - 2\cos 2x$	
	$3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$	
	$3\sin 2x + 4\cos 2x = 2$	A1 (3)
(c)	$3\sin 2x + 4\cos 2x = R\cos(2x - \alpha)$	
	$3\sin 2x + 4\cos 2x = R\cos 2x\cos\alpha + R\sin 2x\sin\alpha$	
	Equate $\sin 2x$: $3 = R\sin\alpha$ Equate $\cos 2x$: $4 = R\cos\alpha$	
	$R = \sqrt{3^2 + 4^2}$;$= \sqrt{25} = 5$	B1
	$\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.86989765...^\circ$	M1 A1
	Hence, $3\sin 2x + 4\cos 2x = 5\cos(2x - 36.87)$	A1 (3)
(d)	$3\sin 2x + 4\cos 2x = 2$	
	$5\cos(2x - 36.87) = 2$	
	$\cos(2x - 36.87) = \frac{2}{5}$	M1
	$(2x - 36.87) = 66.42182...^\circ$	A1
	$(2x - 36.87) = 360 - 66.42182...^\circ$	
	Hence, $x = 51.64591...^\circ$, $165.22409...^\circ$	A1 A1 (4)
		(12 marks)

Question Number	Scheme	Marks
7. (a)	$f(x) = 1 - \dfrac{2}{(x+4)} + \dfrac{x-8}{(x-2)(x+4)} \quad x \in \mathbb{R},\ x \neq -4,\ x \neq 2.$	
	$f(x) = \dfrac{(x-2)(x+4) - 2(x-2) + x - 8}{(x-2)(x+4)}$	M1 A1
	$= \dfrac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$	
	$= \dfrac{x^2 + x - 12}{[(x+4)(x-2)]}$	A1
	$= \dfrac{(x+4)(x-3)}{[(x+4)(x-2)]}$	M1
	$= \dfrac{(x-3)}{(x-2)}$	A1 cso (5)
(b)	$g(x) = \dfrac{e^x - 3}{e^x - 2} \quad x \in \mathbb{R},\ x \neq \ln 2.$	
	Apply quotient rule: $\left\{ \begin{array}{ll} u = e^x - 3 & v = e^x - 2 \\ \dfrac{du}{dx} = e^x & \dfrac{dv}{dx} = e^x \end{array} \right\}$	
	$g'(x) = \dfrac{e^x(e^x - 2) - e^x(e^x - 3)}{(e^x - 2)^2}$	M1 A1
	$= \dfrac{e^x}{(e^x - 2)^2}$	A1 cso (3)
(c)	$g'(x) = 1 \Rightarrow \dfrac{e^x}{(e^x - 2)^2} = 1$	
	$e^x = (e^x - 2)^2$	M1
	$e^x = e^{2x} - 2e^x - 2e^x + 4$	
	$\underline{e^{2x} - 5e^x + 4} = 0$	A1
	$(e^x - 4)(e^x - 1) = 0$	M1
	$e^x = 4$ or $e^x = 1$	
	$x = \ln 4$ or $x = 0$	A1 (4)
		(12 marks)

Question Number	Scheme	Marks
8. (a)	$\sin 2x = \underline{2\sin x \cos x}$	B1 (1)
(b)	$\operatorname{cosec} x - 8\cos x = 0\,, \quad 0 < x < \pi$	
	$\dfrac{1}{\sin x} - 8\cos x = 0$	M1
	$\dfrac{1}{\sin x} = 8\cos x$	
	$1 = 8\sin x \cos x$	
	$1 = 4(2\sin x \cos x)$	
	$1 = 4\sin 2x$	M1
	$\underline{\sin 2x = \tfrac{1}{4}}$	<u>A1</u>
	Radians $2x = \{0.25268..., 2.88891...\}$	
	Degrees $2x = \{14.4775..., 165.5225...\}$	
	Radians $x = \{0.12634..., 1.44445...\}$	A1
	Degrees $x = \{7.23875..., 82.76124...\}$	A1 cao(5)
		(6 marks)