

Mark Scheme (Final)

Summer 2007

GCE

GCE Mathematics (6665/01)

June 2007
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $\ln 3x = \ln 6$ or $\ln x = \ln \left(\frac{6}{3}\right)$ or $\ln \left(\frac{3x}{6}\right) = 0$ $x = 2$ (only this answer)	M1 A1 (cso) (2)
	(b) $(e^x)^2 - 4e^x + 3 = 0$ (any 3 term form) $(e^x - 3)(e^x - 1) = 0$ $e^x = 3$ or $e^x = 1$ Solving quadratic $x = \ln 3, x = 0$ (or $\ln 1$)	M1 M1 dep M1 A1 (4) (6 marks)

Notes: (a) Answer $x = 2$ with no working or no incorrect working seen: M1A1

Note: $x = 2$ from $\ln x = \frac{\ln 6}{\ln 3} = \ln 2$ M0A0

$\ln x = \ln 6 - \ln 3 \Rightarrow x = e^{(\ln 6 - \ln 3)}$ allow M1, $x = 2$ (no wrong working) A1

- (b) 1st M1 for attempting to multiply through by e^x : Allow y, X , even x , for e^x
 2nd M1 is for solving quadratic as far as getting two values for e^x or y or X etc
 3rd M1 is for converting their answer(s) of the form $e^x = k$ to $x = \ln k$ (must be exact)
 A1 is for $\ln 3$ **and** $\ln 1$ or 0 (Both required and no further solutions)

2. (a)	$2x^2 + 3x - 2 = (2x - 1)(x + 2)$ $f(x) = \frac{(2x+3)(2x-1) - (9+2x)}{(2x-1)(x+2)}$ at any stage f.t. on error in denominator factors (need not be single fraction) Simplifying numerator to quadratic form Correct numerator $= \frac{4x^2 + 2x - 12}{[(2x-1)(x+2)]}$ Factorising numerator, with a denominator $= \frac{2(2x-3)(x+2)}{(2x-1)(x+2)}$ o.e. $= \frac{4x-6}{2x-1}$ (*)	B1 M1, A1√ M1 A1 M1 A1 cso (7)
Alt.(a)	$2x^2 + 3x - 2 = (2x - 1)(x + 2)$ $f(x) = \frac{(2x+3)(2x^2 + 3x - 2) - (9+2x)(x+2)}{(x+2)(2x^2 + 3x - 2)}$ $= \frac{4x^3 + 10x^2 - 8x - 24}{(x+2)(2x^2 + 3x - 2)}$ $= \frac{2(x+2)(2x^2 + x - 6)}{(x+2)(2x^2 + 3x - 2)}$ at any stage B1 M1A1 f.t. or $\frac{2(2x-3)(x^2 + 4x + 4)}{(x+2)(2x^2 + 3x + 2)}$ o.e. Any one linear factor × quadratic factor in numerator M1, A1 $= \frac{2(x+2)(x+2)(2x-3)}{(x+2)(2x^2 + 3x - 2)}$ o.e. M1 $= \frac{2(2x-3)}{2x-1} \quad \frac{4x-6}{2x-1}$ (*) A1	
(b)	Complete method for $f'(x)$; e.g. $f'(x) = \frac{(2x-1) \times 4 - (4x-6) \times 2}{(2x-1)^2}$ o.e. $= \frac{8}{(2x-1)^2}$ or $8(2x-1)^{-2}$ Not treating f^{-1} (for f') as misread	M1 A1 A1 (3) (10 marks)

Notes: (a) 1st M1 in either version is for correct method

1st A1 Allow $\frac{2x+3(2x-1) - (9+2x)}{(2x-1)(x+2)}$ or $\frac{(2x+3)(2x-1) - 9 + 2x}{(2x-1)(x+2)}$ or $\frac{2x+3(2x-1) - 9 + 2x}{(2x-1)(x+2)}$ (fractions)

2nd M1 in (main a) is for forming 3 term quadratic in **numerator**

3rd M1 is for factorising their quadratic (usual rules); factor of 2 need not be extracted

(*) A1 is given answer so is cso

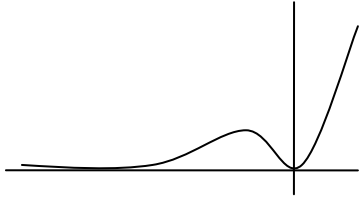
Alt :(a) 3rd M1 is for factorising resulting quadratic

(b) SC: For M allow \pm given expression or one error in product rule

Alt: Attempt at $f(x) = 2 - 4(2x-1)^{-1}$ and diff. M1; $k(2x-1)^{-2}$ A1; A1 as above

Accept $8(4x^2 - 4x + 1)^{-1}$.

Differentiating original function – mark as scheme.

Question Number	Scheme	Marks
3. (a)	$\frac{dy}{dx} = x^2e^x + 2xe^x$	M1,A1,A1 (3)
(b)	If $\frac{dy}{dx} = 0$, $e^x(x^2 + 2x) = 0$ setting $(a) = 0$ $[e^x \neq 0]$ $x(x + 2) = 0$ $(x = 0)$ $x = -2$ $x = 0, y = 0$ and $x = -2, y = 4e^{-2} (= 0.54\dots)$	M1 A1 A1 \surd (3)
(c)	$\frac{d^2y}{dx^2} = x^2e^x + 2xe^x + 2xe^x + 2e^x$ $[= (x^2 + 4x + 2)e^x]$	M1, A1 (2)
(d)	$x = 0, \frac{d^2y}{dx^2} > 0 (=2)$ $x = -2, \frac{d^2y}{dx^2} < 0 [= -2e^{-2} (= -0.270\dots)]$ M1: Evaluate, or state sign of, candidate's (c) for at least one of candidate's x value(s) from (b) \therefore minimum \therefore maximum	M1 A1 (cso) (2)
Alt.(d)	For M1: Evaluate, or state sign of, $\frac{dy}{dx}$ at two appropriate values – on either side of at least one of their answers from (b) or Evaluate y at two appropriate values – on either side of at least one of their answers from (b) or Sketch curve 	

(10 marks)

Notes: (a) M for attempt at $f(x)g'(x) + f'(x)g(x)$

1st A1 for one correct, 2nd A1 for the other correct.

Note that x^2e^x on its own scores no marks

(b) 1st A1 ($x = 0$) may be omitted, but for

2nd A1 both sets of coordinates needed ; f.t only on candidate's $x = -2$

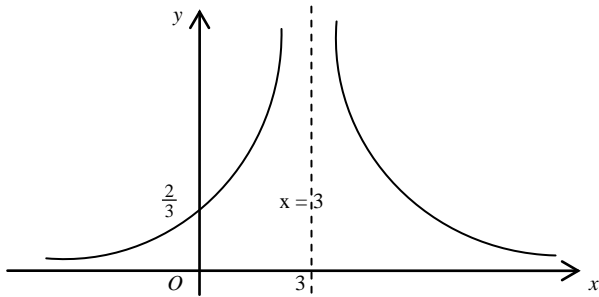
(c) M1 requires complete method for candidate's (a), result may be unsimplified for A1

(d) A1 is cso; $x = 0$, min, and $x = -2$, max and no incorrect working seen,

or (in alternative) sign of $\frac{dy}{dx}$ either side correct, or values of y appropriate to t.p.

Need only consider the quadratic, as may assume $e^x > 0$.

If all marks gained in (a) and (c), and correct x values, give M1A1 for correct statements with no working

Question Number	Scheme	Marks	
4.	(a) $x^2(3-x) - 1 = 0$ o.e. (e.g. $x^2(-x+3) = 1$) $x = \sqrt{\frac{1}{3-x}}$ (*) Note(*), answer is given: need to see appropriate working and A1 is cso [Reverse process: Squaring and non-fractional equation M1, form f(x) A1]	M1 A1 (cso) (2)	
	(b) $x_2 = 0.6455, x_3 = 0.6517, x_4 = 0.6526$ 1 st B1 is for one correct, 2 nd B1 for other two correct If all three are to greater accuracy, award B0 B1	B1; B1 (2)	
	(c) Choose values in interval (0.6525, 0.6535) or tighter and evaluate both $f(0.6525) = -0.0005$ (372... $f(0.6535) = 0.002$ (101... At least one correct "up to bracket", i.e. -0.0005 or 0.002 Change of sign , $\therefore x = 0.653$ is a root (correct) to 3 d.p. Requires both correct "up to bracket" and conclusion as above	M1 A1 A1 (3) (7 marks)	
	Alt (i) Continued iterations at least as far as x_6 M1 $x_5 = 0.65268, x_6 = 0.6527, x_7 = \dots$ two correct to at least 4 s.f. A1 Conclusion : Two values correct to 4 d.p., so 0.653 is root to 3 d.p. A1 Alt (ii) If use $g(0.6525) = 0.6527.. > 0.6525$ and $g(0.6535) = 0.6528.. < 0.6535$ M1A1 Conclusion : Both results correct, so 0.653 is root to 3 d.p. A1		
5.	(a) Finding $g(4) = k$ and $f(k) = \dots$ or $fg(x) = \ln\left(\frac{4}{x-3} - 1\right)$ [$f(2) = \ln(2 \times 2 - 1)$ $fg(4) = \ln(4 - 1)$] = $\ln 3$	M1 A1 (2)	
	(b) $y = \ln(2x-1) \Rightarrow e^y = 2x-1$ or $e^x = 2y-1$ $f^{-1}(x) = \frac{1}{2}(e^x + 1)$ Allow $y = \frac{1}{2}(e^x + 1)$ Domain $x \in \mathcal{R}$ [Allow \mathcal{R} , all reals, $(-\infty, \infty)$] independent	M1, A1 A1 B1 (4)	
	(c) 	Shape, and x-axis should appear to be asymptote Equation $x = 3$ needed , may see in diagram (ignore others) Intercept $(0, \frac{2}{3})$ no other; accept $y = \frac{2}{3}$ (0.67) or on graph	B1 B1 ind. B1 ind (3)
	(d) $\frac{2}{x-3} = 3 \Rightarrow x = 3\frac{2}{3}$ or exact equiv. $\frac{2}{x-3} = -3, \Rightarrow x = 2\frac{1}{3}$ or exact equiv. Note: $2 = 3(x+3)$ or $2 = 3(-x-3)$ o.e. is M0A0 Alt: Squaring to quadratic $(9x^2 - 54x + 77 = 0)$ and solving M1; B1A1	B1 M1, A1 (3) (12 marks)	

6.	(a)	Complete method for R: e.g. $R \cos \alpha = 3, R \sin \alpha = 2, R = \sqrt{3^2 + 2^2}$ $R = \sqrt{13}$ or 3.61 (or more accurate) Complete method for $\tan \alpha = \frac{2}{3}$ [Allow $\tan \alpha = \frac{3}{2}$] $\alpha = 0.588$ (Allow 33.7°)	M1 A1 M1 A1 (4)
	(b)	Greatest value = $(\sqrt{13})^4 = 169$	M1, A1 (2)
	(c)	$\sin(x + 0.588) = \frac{1}{\sqrt{13}}$ (= 0.27735...) $\sin(x + \text{their } \alpha) = \frac{1}{\text{their } R}$ $(x + 0.588) = 0.281(03\dots)$ or 16.1° $(x + 0.588) = \pi - 0.28103\dots$ Must be $\pi - \text{their } 0.281$ or $180^\circ - \text{their } 16.1^\circ$ or $(x + 0.588) = 2\pi + 0.28103\dots$ Must be $2\pi + \text{their } 0.281$ or $360^\circ + \text{their } 16.1^\circ$ $x = 2.273$ or $x = 5.976$ (awrt) Both (radians only) If 0.281 or 16.1° not seen, correct answers imply this A mark	M1 A1 M1 M1 A1 (5) (11 marks)

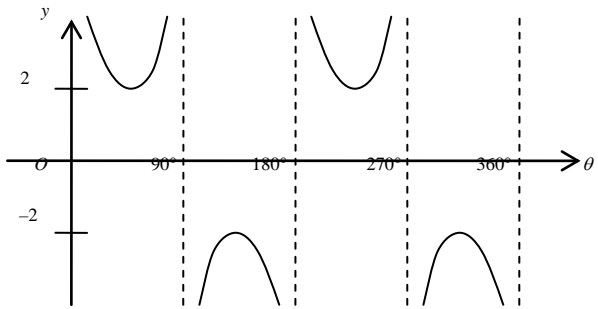
Notes: (a) 1st M1 for correct method for R
 2nd M1 for correct method for $\tan \alpha$
 No working at all: M1A1 for $\sqrt{13}$, M1A1 for 0.588 or 33.7° .
 N.B. $R \cos \alpha = 2, R \sin \alpha = 3$ used, can still score M1A1 for R, but loses the A mark for α .
 $\cos \alpha = 3, \sin \alpha = 2$: apply the same marking.

(b) M1 for realising $\sin(x + \alpha) = \pm 1$, so finding R^4 .

(c) Working in mixed degrees/rads : first two marks available
 Working consistently in degrees: Possible to score first 4 marks
 [Degree answers, just for reference only, are 130.2° and 342.4°]
 Third M1 can be gained for candidate's 0.281 – candidate's $0.588 + 2\pi$ or equiv. in degrees
 One of the answers correct in radians or degrees implies the corresponding M mark.

Alt: (c) (i) Squaring to form quadratic in $\sin x$ or $\cos x$ M1
 $[13 \cos^2 x - 4 \cos x - 8 = 0, 13 \sin^2 x - 6 \sin x - 3 = 0]$
 Correct values for $\cos x = 0.953\dots, -0.646$; or $\sin x = 0.767, 2.27$ awrt A1
 For any one value of $\cos x$ or $\sin x$, correct method for two values of x M1
 $x = 2.273$ or $x = 5.976$ (awrt) Both seen anywhere A1
 Checking other values (0.307, 4.011 or 0.869, 3.449) and discarding M1

(ii) Squaring and forming equation of form $a \cos 2x + b \sin 2x = c$
 $9 \sin^2 x + 4 \cos^2 x + 12 \sin 2x = 1 \Rightarrow 12 \sin 2x + 5 \cos 2x = 11$
 Setting up to solve using R formula e.g. $\sqrt{13} \cos(2x - 1.176) = 11$ M1
 $(2x - 1.176) = \cos^{-1}\left(\frac{11}{\sqrt{13}}\right) = 0.562(0\dots)$ (α) A1
 $(2x - 1.176) = 2\pi - \alpha, 2\pi + \alpha, \dots$ M1
 $x = 2.273$ or $x = 5.976$ (awrt) Both seen anywhere A1
 Checking other values and discarding M1

Question Number	Scheme	Marks
<p>7. (a)</p> <p>Alt.(a)</p>	$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}$ <p>M1 Use of common denominator to obtain single fraction</p> $= \frac{1}{\cos \theta \sin \theta}$ <p>M1 Use of appropriate trig identity (in this case $\sin^2 \theta + \cos^2 \theta = 1$)</p> $= \frac{1}{\frac{1}{2} \sin 2\theta}$ <p>Use of $\sin 2\theta = 2 \sin \theta \cos \theta$</p> $= 2 \operatorname{cosec} 2\theta \quad (*)$ $\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \tan \theta + \frac{1}{\tan \theta} = \frac{\tan^2 \theta + 1}{\tan \theta}$ <p>M1</p> $= \frac{\sec^2 \theta}{\tan \theta}$ <p>M1</p> $= \frac{1}{\cos \theta \sin \theta} = \frac{1}{\frac{1}{2} \sin 2\theta}$ <p>M1</p> $= 2 \operatorname{cosec} 2\theta \quad (*) \quad (\text{cso}) \quad \text{A1}$ <p>If show two expressions are equal, need conclusion such as QED, tick, true.</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1 cso (4)</p>
(b)	 <p>Shape (May be translated but need to see 4 "sections")</p> <p>T.P.s at $y = \pm 2$, asymptotic at correct x-values (dotted lines not required)</p>	<p>B1</p> <p>B1 dep. (2)</p>
(c)	$2 \operatorname{cosec} 2\theta = 3$ $\sin 2\theta = \frac{2}{3} \quad \text{Allow } \frac{2}{\sin 2\theta} = 3 \quad [\text{M1 for equation in } \sin 2\theta]$ <p>(2θ) = [41.810...°, 138.189...°; 401.810...°, 498.189...°]</p> <p>1st M1 for $\alpha, 180 - \alpha$; 2nd M1 adding 360° to at least one of values</p> <p>$\theta = 20.9^\circ, 69.1^\circ, 200.9^\circ, 249.1^\circ$ (1 d.p.) awrt</p> <p>Note 1st A1 for any two correct, 2nd A1 for other two Extra solutions in range lose final A1 only SC: Final 4 marks: $\theta = 20.9^\circ$, after M0M0 is B1; record as M0M0A1A0</p>	<p>M1, A1</p> <p>M1; M1</p> <p>A1, A1 (6)</p>
Alt.(c)	$\tan \theta + \frac{1}{\tan \theta} = 3 \quad \text{and form quadratic, } \tan^2 \theta - 3 \tan \theta + 1 = 0 \quad \text{M1, A1}$ <p>(M1 for attempt to multiply through by $\tan \theta$, A1 for correct equation above)</p> <p>Solving quadratic [$\tan \theta = \frac{3 \pm \sqrt{5}}{2} = 2.618... \text{ or } = 0.3819...$] M1</p> <p>$\theta = 69.1^\circ, 249.1^\circ$ $\theta = 20.9^\circ, 200.9^\circ$ (1 d.p.) M1, A1, A1</p> <p>(M1 is for one use of $180^\circ + \alpha^\circ$, A1A1 as for main scheme)</p>	<p>(12 marks)</p>

Question Number	Scheme	Marks
8. (a)	$D = 10, t = 5, \quad x = 10e^{-\frac{1}{8} \times 5}$ $= 5.353$ awrt	M1 A1 (2)
(b)	$D = 10 + 10e^{-\frac{5}{8}}, t = 1, \quad x = 15.3526... \times e^{-\frac{1}{8}}$ $x = 13.549$ (*)	M1 A1 cso (2)
Alt.(b)	$x = 10e^{-\frac{1}{8} \times 6} + 10e^{-\frac{1}{8} \times 1}$ M1 $x = 13.549$ (*) A1 cso	
(c)	$15.3526...e^{-\frac{1}{8}T} = 3$ $e^{-\frac{1}{8}T} = \frac{3}{15.3526...} = 0.1954...$ $-\frac{1}{8}T = \ln 0.1954...$ $T = 13.06... \text{ or } 13.1 \text{ or } 13$	M1 M1 A1 (3)
		(7 marks)

Notes: (b) (main scheme) M1 is for $(10 + 10e^{-\frac{5}{8}})e^{-\frac{1}{8}}$, or $\{10 + \text{their(a)}\}e^{-\frac{1}{8}}$

N.B. The answer is given. There are many correct answers seen which deserve M0A0
or M1A0

(c) 1st M is for $(10 + 10e^{-\frac{5}{8}})e^{-\frac{T}{8}} = 3$ o.e.

2nd M is for converting $e^{-\frac{T}{8}} = k$ ($k > 0$) to $-\frac{T}{8} = \ln k$. This is independent of 1st M.

Trial and improvement: M1 as scheme,
M1 correct process for their equation (two equal to 3 s.f.)
A1 as scheme